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Preservers of local spectrum of matrix Jordan triple products ☆

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ABSTRACT

Let $\mathcal{M}_n$ be the space of $n \times n$ complex matrices, and for a nonzero vector $e \in \mathbb{C}^n$ and $T \in \mathcal{M}_n$, let $\sigma_T(e)$ denote the local spectrum of T at e. Maps ϕ on $\mathcal{M}_n$ which preserve the local spectrum of Jordan triple product of matrices at e in a sense that

$$\sigma_{\phi(T)\phi(S)\phi(T)}(e) = \sigma_{TST}(e), \quad (T, S \in \mathcal{M}_n)$$

are characterized, with no surjectivity assumption on them.

1. Introduction and statement of the main result

Let X be a complex Banach space over the complex field $\mathbb{C}$. We denote by $\mathcal{L}(X)$ the algebra of all bounded linear operators on X with identity operator I. The local resolvent set of an operator $T \in \mathcal{L}(X)$ at a vector $x \in X$, $\rho_T(x)$, is the set of all λ in $\mathbb{C}$ for which there exists an open neighborhood U_λ of λ in $\mathbb{C}$ and an X-valued analytic

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function $f : U_\lambda \to X$ such that $(\mu - T)f(\mu) = x$ for all $\mu \in U_\lambda$. Its complement in $\mathbb{C}$, denoted by $\sigma_T(x)$, is called the local spectrum of T at x, and is a compact (possibly empty) subset of the classical spectrum $\sigma(T)$ of T. The local spectral radius of T at x is defined by the formula $r_T(x) := \limsup_{n\to+\infty} \|T^n x\|^{\frac{1}{n}}$, and coincides with the maximum modulus of $\sigma_T(x)$ provided that T has the single-valued extension property. Recall that T is said to have the single-valued extension property (or SVEP, for short) if for every open subset U of $\mathbb{C}$, the equation $(\mu - T)f(\mu) = 0$, $(\mu \in U)$, has no nontrivial X-valued analytic solution f on U. Evidently, every operator $T \in \mathcal{L}(X)$ for which the interior of the set of its eigenvalues is empty enjoys this property. In the particular case which we shall consider in this paper, that is when $X = \mathbb{C}^n$ is of finite dimension, every T belonging to $\mathcal{M}_n$, the algebra of $n \times n$ complex matrices, has the SVEP.

Local spectra play a very natural role in automatic continuity and in harmonic analysis, for instance in connection with the Wiener–Pitt phenomenon. For further information on the local spectral theory, as well as investigations and applications in numerous fields, we refer to the books [1,18,22].

Recently, general preserver problems with respect to various algebraic operations on $\mathcal{M}_n$ or on operator algebras, attracted a lot of attention of researchers in the fields. In [12], Dolinar et al. classified mappings on $\mathcal{M}_n$ that compress the spectrum of the difference of matrices. Bhatia et al. [7] characterized surjective maps ϕ on $\mathcal{M}_n$ that preserve the spectral radius distance (i.e., $\phi(T) - \phi(S)$ and $T - S$ always have the same spectral radius for any matrices T and S). For a given real number c, Li et al. [19] studied bijective maps ϕ acting on some subsets of matrices satisfying $tr(TS) = c \Longleftrightarrow tr(\phi(T)\phi(S)) = c$. In [9], Chan et al. characterized mappings on $\mathcal{M}_n$ that preserve the spectrum of different type of products of matrices, including the Jordan triple product. Certain products preservers on operator algebras are considered in [11,14,16,17,20,21,23,25]. On the subject focused on the structures of nonlinear transformations on $\mathcal{M}_n$ or on $\mathcal{L}(X)$ that respect the local spectra of certain algebraic operations, we mention: [3,4] where the authors studied mappings on $\mathcal{M}_n$ that compress (expand) the local spectrum of matrix sums (differences) or that preserve the local spectral radius distance, Costara [10] concerned with local spectral radius distance, and Bendaoud et al. [5,6] treated general preserver problems dealing with preservers of local invertibility and of local spectra of matrix and operator products, and investigation of several extensions of these results were obtained. The corresponding problem for the Jordan triple product was considered by Bourhim and Mashreghi [8, Main result]. Fixing a pair of infinite-dimensional Banach spaces X and Y and a pair of nonzero vectors $x_0 \in X$ and $y_0 \in Y$, they proved that if $\phi : \mathcal{L}(X) \to \mathcal{L}(Y)$ is surjective and

$$\sigma_{\phi(T)\phi(S)\phi(T)}(y_0) = \sigma_{TST}(x_0) \quad \bigl(T, S \in \mathcal{L}(X)\bigr),$$

then there exists a third root of unity ε and an invertible bounded linear operator $A : X \to Y$ such that $Ax_0 = y_0$ and $\phi(T) = \varepsilon ATA^{-1}$ for every $T \in \mathcal{L}(X)$.