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On the periodicity of non-homogeneous Markov chains and systems

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ABSTRACT

We firstly recall the concept of an NHMS and introduce concepts and known results necessary for the study of asymptotic periodicity of the vector of expectations, variances and covariances of the state sizes. This sequence of vectors is called *the variability of population structures*. We then proceed to prove some useful propositions and lemmas for the establishment of the basic results that follow. Note, that some of these lemmas have a power and usefulness of their own. Finally, we provide two theorems, where we study the asymptotic periodicity of the variability of population structures, when the inherent non-homogeneous Markov chain is uniquely determined by a sequence of arbitrary stochastic matrices all with the same incidence matrix. Following that an illustrative example from the area of manpower systems is provided. In this example the results in the basic theorems of the previous section are illustrated numerically. We conclude with an appendix, where we firstly introduce the necessary concepts and known results, needed in order to study the periodicity and asymptotic periodicity of an infinite product of finite arbitrary stochastic matrices. We then proceed to study the periodicity and asymptotic periodicity of non-homogeneous Markov chains in their full possible generality.

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It is very important to note that similar results hold also for an infinite product of non-negative matrices.

1. Introductory notes

The problem of periodicity and asymptotic periodicity is central in the theory of non-homogeneous Markov chains and also non-homogeneous Markov systems (NHMS). Naturally, it was also central from the start in the simpler case of the theory of homogeneous Markov chains (Seneta [26], Hunter [8,9]). In the non-homogeneous Markov chain case, when the state space is finite or countable, the basic results were provided by Isaacson [13], Bowerman et al. [1] and Iosifescu [12]. The NHMS is a population stochastic process, which was first defined in Vassiliou [33]. Its roots are many manpower systems reported in the literature and a compact first review of the theory, that since has flourished considerably, exists in Vassiliou [35]. For the interested reader a second review exists in Ugwugo and McClean [40]. In the general framework of an NHMS a great variety of applied probability models could be accommodated. Applications of the theory could only be referred selectively due to the large number of such cases. Hence, as examples we refer to some recent applications such as, the evolution of the population of the HIV virus within the human system of T-cells in Mathew et al. [15] and Foucher et al. [5]; in the disease of asthma in Saint Pierre [25]; in reliability, for example Limnios and Oprisan [14]; in general biomedical studies (Perez-Ocon and Castro [21] and Ocana-Riola [19]); in gene expression sequences (McClean et al. [17]); in manpower systems in De Freyter [3], Nilakantan and Raghavendra [18], Yadavalli et al. [41], Sen Gupta and Ugwuogo [27], De Freyter and Guerry [4], Guerry [11]; in Physical Chemistry (Crooks [2]); and in ecology (Patoucheas and Stamou [20]). For NHMS the majority of results for the periodicity and asymptotic periodicity are contained in Vassiliou [33], Tsaklidis and Vassiliou [29] and Georgiou and Vassiliou [10]. Periodicity and asymptotic periodicity has been studied also for Markov chains in discrete time and general state space (Meyn [16]) and in discrete and countable space in Vassiliou [38]. The study of periodicity and asymptotic periodicity for Markov systems in discrete time and general state space exists in Vassiliou [37]. In all the above studies of non-homogeneous Markov chains and systems in discrete time and finite or countable state space, the basic assumption was that all the matrices in the sequence of transition probability matrices were regular and periodic. That is, every matrix in the sequence of transition probability matrices is irreducible and consists of one essential class of states, which is periodic. In the simple Markov chain case this was also the basic assumption in all the textbooks. However, in the paper by Sierksma [28] the periodicity and asymptotic periodicity were studied under the assumption of an arbitrary stochastic matrix for the homogeneous Markov chain in discrete time and finite state space. Therefore, the study of periodicity and asymptotic periodicity for the homogeneous Markov chain has been taken an important