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Estimation of parameters in the extended growth curve model via outer product least squares for covariance

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ABSTRACT

In this paper, estimation of parameters in the extended growth curve model without assumption of normality is presented via outer product least squares for covariance (COPLS). The COPLS estimator for covariance can be explicitly expressed and proved to follow a linear transformation of $k+1$ independent Wishart distributions for a normal error matrix, where k is the number of profiles of the growth curve. Then two-stage generalized least squares (GLS) estimators for regression coefficients are elaborately derived. Finally, the COPLS estimator and the two-stage GLS estimators are shown to have desired properties both in finite and large samples. Simulation studies for finite sample sizes are provided to confirm that the COPLS estimator and the two-stage GLS estimators are alternative competitors with some evident merits to the existing maximum likelihood estimators in small samples. A real data set is analyzed to demonstrate the proposed methodology.

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1. Introduction

A classical growth curve model or a generalized multivariate analysis-of-variance model was initiated by Potthoff and Roy [16] and widely investigated by many scholars, among them, including [2,5,7,10–13,15,17,18,26]. This model is especially useful for analyzing growth (curve) problems on short time series in areas such as biological science, economics, medical research and epidemiology in which observations are usually measured over multiple time points on a characteristic to analyze temporal pattern of change on the characteristic. It is also a fundamental tool for conducting longitudinal data analysis especially with serial correlation [9].

When some hypothesis tests of the regression coefficients [21] or response curves concurrent at these multiple time points or covariates for responses [20] are taken into account, we may postulate the extended growth curve model, namely,

$$\mathbf{Y} = X_1\Theta_1Z_1' + X_2\Theta_2Z_2' + \cdots + X_k\Theta_kZ_k' + \mathcal{E} \tag{1.1}$$

subject to nested constraints

$$\mathrm{r}(X_1) + p \le n \quad \text{and} \quad \mathscr{C}(X_1) \supseteq \mathscr{C}(X_2) \supseteq \cdots \supseteq \mathscr{C}(X_k), \tag{1.2}$$

where $\mathbf{Y}$ is the $n \times p$ observation matrix of the response consisting of p repeated measurements taken on n individuals, X_i is the ith treatment design matrix with order $n \times m_i$, Z_i is the ith profile matrix with order $p \times q_i$, Θ_i is the ith unknown regression coefficient matrix with order $m_i \times q_i$, and $\mathrm{r}(X)$ and $\mathscr{C}(X)$ denote the rank and the column space of the matrix X, respectively. Assume that observations on individuals are independent, so that the rows of the random error matrix $\mathcal{E}$ are independent and identically distributed by a general continuous type distribution $\mathcal{F}$ with mean zero and a common covariance matrix Σ of order p.

Several authors studied the estimating problem for the extended growth curve model. von Rosen [21] presented maximum likelihood estimators. Wu [23] gave an existence condition for the uniformly minimum risk unbiased estimators. Wu, Zou and Chen [24] investigated unbiased invariant minimum norm and uniformly minimum variance nonnegative quadratic unbiased estimators for a linear parameter function $\mathrm{tr}(C\Sigma)$ of Σ. Wu, Liang and Zou [22] obtained the necessary and sufficient conditions of the unbiased invariant least squares estimators for the linear function $\mathrm{tr}(C\Sigma)$ of Σ. Ye and Wang [25] derived estimators of parameters when covariance has a special covariance structure. In addition, Hamid and von Rosen [3] analyzed the residuals.

The above mentioned researches are based on assumption of normality of errors. However, no work has been done on estimators and their asymptotic properties for parameters in the extended growth curve model without assumption of normality. Recently, Hu, Liu and Ahmed [5] combined outer product least squares for covariance (COPLS) and two-stage generalized least squares (GLS) to investigate asymptotic properties of the COPLS