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Non-bipartite graphs of fixed order and size that minimize the least eigenvalue

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ABSTRACT

In this paper we determine the unique graph with minimal least eigenvalue (of the adjacency matrix) within the set of connected graphs of fixed order n and size m, whenever $m = \lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor + a$, where a is a fixed integral constant in $[1, \lceil \frac{n}{2} \rceil - 1]$.

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1. Introduction

Let G be a simple graph with vertex set V_G and edge set E_G. Its order $|V_G|$ and size $|E_G|$ are denoted by n and m. The characteristic polynomial (of the adjacency matrix A_G) of G is denoted by φ_G, while the corresponding roots,

$$\lambda_1(G) \geq \lambda_2(G) \geq \cdots \geq \lambda_n(G),$$

are just the *eigenvalues* of G. In this paper the least eigenvalue is rather denoted by $\lambda(G)$. The collection of eigenvalues (with repetition) is called the *spectrum* of G.

In comparison with the largest one, the least eigenvalue has received less attention in literature and the most of results are dealt with producing bounds or characterizing graphs with minimal least eigenvalue subject to prescribed properties (not to be listed here).

The investigation of connected graphs of fixed order and size that minimize the least eigenvalue is initiated by Bell et al. [1,2]. They proved that any such a graph is either bipartite or obtained as a join of two nested split graphs (the definitions are given in the next section).

Since for bipartite graphs $\lambda_1(G) = -\lambda(G)$, the investigation of connected bipartite graphs with minimal least eigenvalue reduces to the investigation of the same graphs with maximal largest eigenvalue. Such investigation is introduced by Cvetković et al. [4] and rediscovered later by Bhattacharya et al. [3]. It is proved that the resulting graph must belong to the class of so-called double nested graphs. The problem is resolved in certain particular cases in which the connectedness is not stipulated and as the result appears a graph obtained from a complete bipartite graph by joining some vertices that belong to one colour class with an isolated vertex. It is also conjectured that any solution to this problem must be of this kind. The subject of this paper are connected non-bipartite graphs and, as it is announced in the abstract, we determine the unique graph (of fixed order and size) that minimizes the least eigenvalue whenever its size equals $\lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor + a$, where a is a fixed constant in $\left[1, \lceil \frac{n}{2} \rceil - 1\right]$.

The paper is organized as follows. In Section 2 we fix the basic notation, give preliminary results and set up some details that will be frequently used in the sequel. In Section 3 we formulate our main result and, to make the reading easier, give a brief concept of its proof. The considerably long proof with many details is separated into Section 4.

2. Preliminaries

A path, cycle, totally disconnected and a complete graph on n vertices are denoted by P_n, C_n, O_n and K_n, respectively. A complete bipartite graph with n_1 (resp. n_2) vertices in the first (resp. second) colour class is denoted by K_{n_1,n_2}. Any complete (resp. totally disconnected) induced subgraph of a graph is called the clique (resp. co-clique).