

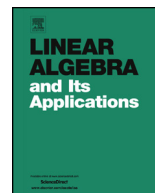


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## Some generalized numerical radius inequalities for Hilbert space operators



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### ABSTRACT

We generalize several inequalities involving powers of the numerical radius for product of two operators acting on a Hilbert space. For any  $A, B, X \in \mathbb{B}(\mathcal{H})$  such that  $A, B$  are positive, we establish some numerical radius inequalities for  $A^\alpha X B^\alpha$  and  $A^\alpha X B^{1-\alpha}$  ( $0 \leq \alpha \leq 1$ ) and Heinz means under mild conditions.

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### 1. Introduction

Let  $(\mathcal{H}, \langle \cdot, \cdot \rangle)$  be a complex Hilbert space and  $\mathbb{B}(\mathcal{H})$  denote the  $C^*$ -algebra of all bounded linear operators on  $\mathcal{H}$ . An operator  $A \in \mathbb{B}(\mathcal{H})$  is called positive if  $\langle Ax, x \rangle \geq 0$  for all  $x \in \mathcal{H}$ . We write  $A \geq 0$  if  $A$  is positive. The numerical radius of  $A \in \mathbb{B}(\mathcal{H})$  is defined by

$$w(A) = \sup\{|\langle Ax, x \rangle| : x \in \mathcal{H}, \|x\| = 1\}.$$

It is well known that  $w(\cdot)$  defines a norm on  $\mathbb{B}(\mathcal{H})$ , which is equivalent to the usual operator norm  $\|\cdot\|$ . In fact, for any  $A \in \mathbb{B}(\mathcal{H})$ ,

$$\frac{1}{2}\|A\| \leq w(A) \leq \|A\|. \tag{1.1}$$

Also if  $A \in \mathbb{B}(\mathcal{H})$  is self-adjoint, then  $w(A) = \|A\|$ .

An important inequality for  $w(A)$  is the power inequality stating that

$$w(A^n) \leq w^n(A)$$

for  $n = 1, 2, \dots$

Several numerical radius inequalities improving the inequalities in (1.1) have been recently given in [2,3,8].

For instance, Dragomir proved that for any  $A, B \in \mathbb{B}(\mathcal{H})$ ,

$$w^2(A) \leq \frac{1}{2}(w(A^2) + \|A\|^2), \tag{1.2}$$

and

$$w^r(B^*A) \leq \frac{1}{2}\|(A^*A)^r + (B^*B)^r\| \tag{1.3}$$

for all  $r \geq 1$ . The above inequalities can be found in [3,1], respectively. Some other interesting inequalities for numerical radius can be found in [8–10].

In Section 2 of this paper, we first generalize inequalities (1.2) and (1.3). Our generalization of inequality (1.3) in a particular case is sharper than this inequality.

In Section 3 we obtain numerical radius inequalities for Hilbert space operators  $A^\alpha X B^\alpha$  and  $A^\alpha X B^{1-\alpha}$  under conditions  $A, B \geq 0$  and  $0 \leq \alpha \leq 1$ . We also find a numerical radius inequality for Heinz means.

### 2. Numerical radius inequalities for products of two operators

To prove our generalized numerical radius inequalities, we need several well-known lemmas. The first lemma is a simple consequence of the classical Jensen and Young inequalities (see [5]).

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