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Stability of reducing subspaces ☆

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We characterize the stability of reducing subspaces of a rectangular matrix pencil of complex matrices $\lambda B - A$, except for the special case in which the pencil has no eigenvalues and only has one row and one column minimal indices and both are different from zero.

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Deflating subspaces
Stability
Perturbation

1. Introduction

Given two matrices $A, B \in \mathbb{C}^{m\times n}$, we call *matrix pencil* the first order matrix polynomial $\lambda B - A$. For simplicity, we will denote the set of matrix pencils of the form $\lambda B - A$, with $A, B \in \mathbb{C}^{m\times n}$, by $\mathcal{P}[\lambda]^{m\times n}$. We define the *normal rank* of a pencil $\lambda B - A \in \mathcal{P}[\lambda]^{m\times n}$, and we denote it by $\operatorname{nrank}(\lambda B - A)$, to be the greatest order of the minors of $\lambda B - A$ that are different from the zero polynomial. If $m = n$ and $\operatorname{nrank}(\lambda B - A) = n$, the pencil $\lambda B - A \in \mathcal{P}[\lambda]^{m\times n}$ is said to be *regular*. Otherwise, the pencil is said to be *singular*.

Note by $\mathbb{C}(\lambda)$ the field of rational fractions in λ. If we consider $\lambda B - A$ as a linear map from the vector space $\mathbb{C}(\lambda)^n$ into $\mathbb{C}(\lambda)^m$, both over the field $\mathbb{C}(\lambda)$, we have

$$\operatorname{nrank}(\lambda B - A) = \dim_{\mathbb{C}(\lambda)} \operatorname{Im}(\lambda B - A),$$

(see [4]). We define the nullity of $\lambda B - A$ by $\nu(\lambda B - A) := \dim_{\mathbb{C}(\lambda)} \operatorname{Ker}(\lambda B - A)$. From

$$\begin{aligned} n &= \dim_{\mathbb{C}(\lambda)} \operatorname{Ker}(\lambda B - A) + \dim_{\mathbb{C}(\lambda)} \operatorname{Im}(\lambda B - A), \\ \nu(\lambda B - A) &= n - \operatorname{nrank}(\lambda B - A). \end{aligned}$$

As usual we identify a matrix $M \in \mathbb{C}^{m\times n}$ with the linear map $x \mapsto Mx$ from $\mathbb{C}^n \equiv \mathbb{C}^{n\times 1}$ into $\mathbb{C}^m \equiv \mathbb{C}^{m\times 1}$. Let $\mathcal{N}$ be a subspace of $\mathbb{C}^n$, we define $M(\mathcal{N})$ as the subspace of $\mathbb{C}^m$ formed by all matrix products Mx with $x \in \mathcal{N}$. Van Dooren proved that

$$\dim\bigl(A(\mathcal{N}) + B(\mathcal{N})\bigr) \geq \dim \mathcal{N} - \nu(\lambda B - A), \tag{1}$$

where $A(\mathcal{N}) + B(\mathcal{N})$ is the sum of these subspaces of $\mathbb{C}^m$. See [17, Eq. (2.16) on page 63 and in the line following (2.25a) and (2.25b) on page 65]. In the case the equality holds in (1), the subspace $\mathcal{N}$ is called a *reducing subspace* for the pencil (see [17]) or, equivalently, that $\mathcal{N}$ is a $(\lambda B - A)$-*reducing subspace*. Observe that if the pencil is regular then $\nu(\lambda B - A) = 0$. So, in this case $\mathcal{N}$ is a reducing subspace if and only if $\dim(A(\mathcal{N}) + B(\mathcal{N})) = \dim \mathcal{N}$. These reducing subspaces are also called *deflating subspaces* for regular pencils (see [16]). In order to simplify here on, we will denote by $(\lambda B - A)(\mathcal{N})$ the subspace $A(\mathcal{N}) + B(\mathcal{N})$.

We use the operator norm induced by the Euclidean norms on $\mathbb{C}^m$ and $\mathbb{C}^n$, also called the spectral norm,

$$\|M\| := \max_{\substack{x \in \mathbb{C}^n \\ \|x\|_2 = 1}} \|Mx\|_2.$$