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The dimension of magic squares over fields of characteristics two and three

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A R T I C L E I N F O

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A B S T R A C T

Hou et al. [4] have studied various spaces of magic squares over a field F and determined their dimensions. However, they left one open question unsolved, namely, if the characteristic of F is 2 or 3, exactly which n and k make $\mathcal{M}_{n,k}(1)$ nonempty, where $\mathcal{M}_{n,k}(1)$ denotes the set of all $n \times n$ matrices over F whose row sums, column sums, k diagonal sums, and k antidiagonal sums are all 1. We solve this completely.

1. Introduction

A *magic square* is an $n \times n$ array of numbers whose rows, columns and the two main diagonals all have the same sum. The study of magic squares has a very old history. The first magic square is known to be found in a Chinese literature, back in the pre-Christ

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era, and this spread from China to India and then to the Arab countries. From Arab countries, the magic squares were introduced to Europe [1,2]. Since the time when the magic squares were recognized as one of the mathematical areas, there have been a lot of studies about this interesting subject [1,3–8].

In this paper, we deal with magic squares over a field. A *magic square of size* n *and weight* d *over a field* F is a matrix (a_{ij}) $(1 \le i, j \le n)$ with entries in F satisfying the following conditions:

$$\sum_{j=1}^{n} a_{ij} = d \qquad \text{for all } 1 \le i \le n \text{ (row sums)}$$

$$\sum_{i=1}^{n} a_{ij} = d \qquad \text{for all } 1 \le j \le n \text{ (column sums)}$$

$$\sum_{i-j=0} a_{ij} = d \qquad \text{(diagonal sum)}$$

$$\sum_{i+j=n+1} a_{ij} = d \qquad \text{(antidiagonal sum)}$$

where $n \in \mathbb{N}$ and $d \in F$. The set of all magic squares of size n over a field F forms a vector space. In [7], Small found the dimension of the vector space of all magic squares of size n. In fact, the same result was founded in [3,5,6,8] in different ways.

In [4], the authors generalized the concept of the magic square over a field in the following manner. Let F be a field, $d \in F$, $n \in \mathbb{N}$, and $0 \le k \le n$. Let $\mathcal{M}_{n,k}(d)$ be the set of all $n \times n$ matrices (a_{ij}) $(1 \le i, j \le n)$ with entries in F satisfying the following conditions:

$$r_i := \sum_{j=1}^{n} a_{ij} = d \qquad \text{for all } 1 \le i \le n \text{ (row sums)}$$

$$c_j := \sum_{i=1}^{n} a_{ij} = d \qquad \text{for all } 1 \le j \le n \text{ (column sums)}$$

$$d_l := \sum_{i-j=-l+1} a_{ij} = d \qquad \text{for all } 1 \le l \le k \text{ (diagonal sums)} \quad \text{if } k \ge 1$$

$$ad_l := \sum_{i+j=-l+2} a_{ij} = d \qquad \text{for all } 1 \le l \le k \text{ (antidiagonal sums)} \quad \text{if } k \ge 1$$

where the subscripts are taken modulo n. For example, d_l and ad_l are described as follows.