

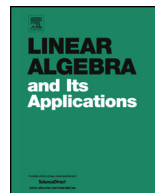


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# The dimension of magic squares over fields of characteristics two and three



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## ABSTRACT

Hou et al. [4] have studied various spaces of magic squares over a field  $F$  and determined their dimensions. However, they left one open question unsolved, namely, if the characteristic of  $F$  is 2 or 3, exactly which  $n$  and  $k$  make  $\mathcal{M}_{n,k}(1)$  nonempty, where  $\mathcal{M}_{n,k}(1)$  denotes the set of all  $n \times n$  matrices over  $F$  whose row sums, column sums,  $k$  diagonal sums, and  $k$  anti-diagonal sums are all 1. We solve this completely.

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## 1. Introduction

A *magic square* is an  $n \times n$  array of numbers whose rows, columns and the two main diagonals all have the same sum. The study of magic squares has a very old history. The first magic square is known to be found in a Chinese literature, back in the pre-Christ

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era, and this spread from China to India and then to the Arab countries. From Arab countries, the magic squares were introduced to Europe [1,2]. Since the time when the magic squares were recognized as one of the mathematical areas, there have been a lot of studies about this interesting subject [1,3–8].

In this paper, we deal with magic squares over a field. A *magic square of size  $n$  and weight  $d$  over a field  $F$*  is a matrix  $(a_{ij})$  ( $1 \leq i, j \leq n$ ) with entries in  $F$  satisfying the following conditions:

$$\begin{aligned} \sum_{j=1}^n a_{ij} &= d && \text{for all } 1 \leq i \leq n \text{ (row sums)} \\ \sum_{i=1}^n a_{ij} &= d && \text{for all } 1 \leq j \leq n \text{ (column sums)} \\ \sum_{i-j=0} a_{ij} &= d && \text{(diagonal sum)} \\ \sum_{i+j=n+1} a_{ij} &= d && \text{(antidiagonal sum)} \end{aligned}$$

where  $n \in \mathbb{N}$  and  $d \in F$ . The set of all magic squares of size  $n$  over a field  $F$  forms a vector space. In [7], Small found the dimension of the vector space of all magic squares of size  $n$ . In fact, the same result was founded in [3,5,6,8] in different ways.

In [4], the authors generalized the concept of the magic square over a field in the following manner. Let  $F$  be a field,  $d \in F$ ,  $n \in \mathbb{N}$ , and  $0 \leq k \leq n$ . Let  $\mathcal{M}_{n,k}(d)$  be the set of all  $n \times n$  matrices  $(a_{ij})$  ( $1 \leq i, j \leq n$ ) with entries in  $F$  satisfying the following conditions:

$$\begin{aligned} r_i &:= \sum_{j=1}^n a_{ij} = d && \text{for all } 1 \leq i \leq n \text{ (row sums)} \\ c_j &:= \sum_{i=1}^n a_{ij} = d && \text{for all } 1 \leq j \leq n \text{ (column sums)} \\ d_l &:= \sum_{i-j=-l+1} a_{ij} = d && \text{for all } 1 \leq l \leq k \text{ (diagonal sums) if } k \geq 1 \\ ad_l &:= \sum_{i+j=-l+2} a_{ij} = d && \text{for all } 1 \leq l \leq k \text{ (antidiagonal sums) if } k \geq 1 \end{aligned}$$

where the subscripts are taken modulo  $n$ . For example,  $d_l$  and  $ad_l$  are described as follows.

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