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The largest Laplacian and signless Laplacian H-eigenvalues of a uniform hypergraph

Shenglong Hu [a,b,1], Liqun Qi [b,2], Jinshan Xie [c,*,3]

[a] *Department of Mathematics, School of Science, Tianjin University, Tianjin, 300072, China*
[b] *Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong*
[c] *School of Mathematics and Computer Science, Longyan University, Longyan, Fujian, China*

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ABSTRACT

In this paper, we show that the largest Laplacian H-eigenvalue of a k-uniform nontrivial hypergraph is strictly larger than the maximum degree when k is even. A tight lower bound for this eigenvalue is given. For a connected even-uniform hypergraph, this lower bound is achieved if and only if it is a hyperstar. However, when k is odd, in certain cases the largest Laplacian H-eigenvalue is equal to the maximum degree, which is a tight lower bound. On the other hand, tight upper and lower bounds for the largest signless Laplacian H-eigenvalue of a k-uniform connected hypergraph are given. For connected k-uniform hypergraphs of fixed number of vertices (respectively fixed maximum degree), the upper (respectively lower) bound of their largest signless Laplacian H-eigenvalues is achieved exactly for the complete hypergraph (respectively the hyperstar). The largest Laplacian H-eigenvalue is always less than or equal to the

* Corresponding author.
E-mail addresses: timhu@tju.edu.cn, tim.hu@connect.polyu.hk (S. Hu), maqilq@polyu.edu.hk (L. Qi), jinshan0623@sina.com (J. Xie).

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largest signless Laplacian H-eigenvalue. When the hypergraph is connected, the equality holds here if and only if k is even and the hypergraph is odd-bipartite.

1. Introduction

In this paper, we study the largest Laplacian and signless Laplacian H-eigenvalues of a uniform hypergraph. The largest Laplacian and signless Laplacian H-eigenvalues refer to respectively the largest H-eigenvalue of the Laplacian tensor and the largest H-eigenvalue of the signless Laplacian tensor. This work is motivated by some classical results for graphs [4,2,6,28,27]. Please refer to [15,9,5,18,16,13,22,21,24,25,10,3,8,14,17,19,23,26] for recent developments on spectral hypergraph theory and the essential tools from spectral theory of nonnegative tensors.

This work is a companion of the recent study on the eigenvectors of the zero Laplacian and signless Laplacian eigenvalues of a uniform hypergraph by Hu and Qi [11]. For the literature on the Laplacian-type tensors for a uniform hypergraph, which becomes an active research frontier in spectral hypergraph theory, please refer to [9,13,24,18,10,26,11] and references therein. Among others, Qi [18], and Hu and Qi [10] respectively systematically studied the Laplacian and signless Laplacian tensors, and the Laplacian of a uniform hypergraph. These three notions of Laplacian-type tensors are more natural and simpler than those in the literature.

The rest of this paper is organized as follows. Some definitions on eigenvalues of tensors and uniform hypergraphs are presented in the next section. The class of hyperstars is introduced. We discuss in Section 3 the largest Laplacian H-eigenvalue of a k-uniform hypergraph. We show that when k is even, the largest Laplacian H-eigenvalue has a tight lower bound that is strictly larger than the maximum degree. Extreme hypergraphs in this case are characterized, which are the hyperstars. When k is odd, a tight lower bound is exactly the maximum degree. However, we are not able to characterize the extreme hypergraphs in this case. Then we discuss the largest signless Laplacian H-eigenvalue in Section 4. Tight lower and upper bounds for the largest signless Laplacian H-eigenvalue of a connected hypergraph are given. Extreme hypergraphs are characterized as well. For the lower bound, the extreme hypergraphs are hyperstars; and for the upper bound, the extreme hypergraphs are complete hypergraphs. The relationship between the largest Laplacian H-eigenvalue and the largest signless Laplacian H-eigenvalue is discussed in Section 5. The largest Laplacian H-eigenvalue is always less than or equal to the largest signless Laplacian H-eigenvalue. When the hypergraph is connected, the equality holds here if and only if k is even and the hypergraph is odd-bipartite. This result can help us to find the largest Laplacian H-eigenvalue of an even-uniform hypercycle. Some final remarks are made in the last section.