

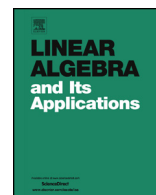


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On the game-theoretic value of a linear transformation relative to a self-dual cone



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ABSTRACT

This paper is concerned with a generalization of the concept of value of a (zero-sum) matrix game. Given a finite dimensional real inner product space V with a self-dual cone K , an element e in the interior of K , and a linear transformation L , we define the value of L by

$$v(L) := \max_{x \in \Delta} \min_{y \in \Delta} \langle L(x), y \rangle = \min_{y \in \Delta} \max_{x \in \Delta} \langle L(x), y \rangle,$$

where $\Delta = \{x \in K : \langle x, e \rangle = 1\}$. This reduces to the classical value of a square matrix when $V = \mathbb{R}^n$, $K = \mathbb{R}_+^n$, and e is the vector of ones. In this paper, we extend some classical results of Kaplansky and Raghavan to this general setting. In addition, for a **Z**-transformation (which is a generalization of a **Z**-matrix), we relate the value with various properties such as the positive stable property, the **S**-property, etc. We apply these results to find the values of the Lyapunov transformation L_A and the Stein transformation S_A on the cone of $n \times n$ real symmetric positive semidefinite matrices.

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1. Introduction

This paper is concerned with a generalization of the concept of value of a (zero-sum) matrix game. To explain, we consider an $n \times n$ real matrix A and the strategy set $X := \{x \in R_+^n : \sum_1^n x_i = 1\}$, where R_+^n denotes the nonnegative orthant in R^n . Then the value of A is given by

$$v(A) := \max_{x \in X} \min_{y \in X} \langle Ax, y \rangle = \min_{y \in X} \max_{x \in X} \langle Ax, y \rangle,$$

where $\langle Ax, y \rangle$ denotes the (usual) inner product between vectors Ax and y . Corresponding to this, there exist *optimal* strategies $\bar{x}, \bar{y} \in X$ such that

$$\langle Ax, \bar{y} \rangle \leq v(A) = \langle A\bar{x}, \bar{y} \rangle \leq \langle A\bar{x}, y \rangle \quad \forall x, y \in X.$$

The concept of value of a matrix and its applications are classical and have been well studied and documented in the game theory literature; see, for example, [12,13]. Our motivation for the generalization comes from results of Kaplansky and Raghavan. In [11], Kaplansky defines a completely mixed (matrix) game as one in which $\bar{x} > 0$ and $\bar{y} > 0$ for every pair of optimal strategies (\bar{x}, \bar{y}) . For such a game, Kaplansky proves the uniqueness of the optimal strategy pair. In [14], Raghavan shows that for a **Z**-matrix (which is a square matrix whose off-diagonal entries are all non-positive) the game is completely mixed when the value is positive, and relates the property of value being positive to a number of equivalent properties of the matrix such as the positive stable property, the **P**-property, etc. His result, in particular, says that for a **Z**-matrix A , the value is positive if and only if there exists an $\bar{x} \in R^n$ such that

$$\bar{x} > 0 \quad \text{and} \quad A\bar{x} > 0.$$

Inequalities of the above type also appear in the study of linear continuous and discrete dynamical systems: Given an $n \times n$ real matrix A , the continuous dynamical system $\frac{dx}{dt} + Ax(t) = 0$ is asymptotically stable on R^n (which means that any trajectory starting from an arbitrary point in R^n converges to the origin) if and only if there exists a real symmetric matrix \bar{X} such that

$$\bar{X} > 0 \quad \text{and} \quad L_A(\bar{X}) > 0,$$

where $\bar{X} > 0$ means that \bar{X} is positive definite, etc., and L_A denotes the so-called *Lyapunov transformation* defined on the space \mathcal{S}^n of all $n \times n$ real symmetric matrices:

$$L_A(X) := AX + XA^T \quad (X \in \mathcal{S}^n).$$

Similarly, the discrete dynamical system $x(k+1) = Ax(k)$, $k = 0, 1, \dots$, is asymptotically stable on R^n if and only if there exists a real symmetric matrix \bar{X} such that

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