

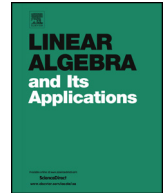


ELSEVIER

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa



Rank structure properties of rectangular matrices admitting bidiagonal-type factorizations



Rong Huang¹

*School of Mathematics and Computational Science, Xiangtan University,
Xiangtan 411105, Hunan, China*

ARTICLE INFO

Article history:

Received 13 June 2014

Accepted 8 September 2014

Available online 25 September 2014

Submitted by R. Brualdi

MSC:

15A23

15A57

Keywords:

Bidiagonal factorization

Neville elimination

Rank structure

ABSTRACT

In this paper, we investigate the class of rectangular matrices that admit bidiagonal-type factorizations by Neville elimination without exchanges. We provide a complete characterization for a rectangular matrix to be factored as a product of bidiagonal factors and a banded factor in terms of rank structure properties. Consequently, we give a complete characterization of the class of rectangular matrices that have Neville factorizations.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Factorizations of matrices as a product of bidiagonal-type factors have played an important role in many applications. For example, it is well-known that a nonsingular totally nonnegative matrix (i.e., all its minors are nonnegative) can be factored as a product of nonnegative bidiagonal factors [9], and thus, by virtue of the bidiagonal

E-mail address: rongh98@aliyun.com.

¹ This work was supported by the National Natural Science Foundation of China (Grant No. 11471279), the Science and Technology Program of Hunan Province (Grant No. 2014FJ3087) and the Research Foundation of Education Bureau of Hunan Province (Grant No. 14B178).

factorization, high accurate computations have been successfully performed including computing its inverse, LDU factorization, eigenvalues and SVD [13,14]. The existence and uniqueness of the bidiagonal factorization is critical to these high accurate computations. Because of the importance, much work has been devoted to bidiagonal-type factorizations of matrices and related numerical computations [1,2,4,5,10,12,15]. As has been known, Neville elimination is a very effective tool to derive the bidiagonal-type factorization.

Neville elimination is a classical elimination technique which, differently from Gaussian elimination, produces zeros in a column by subtracting from each row an adequate multiple of the previous one. This elimination method has been precisely described and extensively studied in [6–8,11]. Next let us illustrate the elimination method by reducing a matrix into a banded matrix including a diagonal matrix.

Definition 1.1. Let $A = (a_{ij}) \in \mathbb{C}^{n \times m}$, and let $0 \leq p \leq n - 1$ and $0 \leq q \leq m - 1$. If $a_{ij} = 0$ for all $i - j > p$, then A is a lower p -banded matrix; if $a_{ij} = 0$ for all $j - i > q$, then A is an upper q -banded matrix; if A is lower p -banded and upper q -banded, then A is called a (p, q) -banded matrix.

For a matrix $A = (a_{ij}) \in \mathbb{C}^{n \times m}$, Neville elimination reduces the matrix A into a lower p -banded matrix in the following procedure:

$$\begin{aligned} A &= \tilde{A}^{(1)} \rightarrow A^{(1)} \rightarrow \tilde{A}^{(2)} \rightarrow \dots \rightarrow \tilde{A}^{(r)} \rightarrow A^{(r)} \\ &\rightarrow \tilde{A}^{(r+1)} = A^{(r+1)} = U, \quad r = \min\{n - p - 1, m\}, \end{aligned}$$

where $U \in \mathbb{C}^{n \times m}$ is a lower p -banded matrix, and for all $1 \leq t \leq r$, $A^{(t)} = (a_{ij}^{(t)})$ is obtained from $\tilde{A}^{(t)}$ by reordering the rows $t + p, \dots, n$ such that

$$a_{it}^{(t)} = 0, \quad i \geq t + p \quad \Rightarrow \quad a_{ht}^{(t)} = 0, \quad \forall h \geq i; \quad (1.1)$$

$\tilde{A}^{(t+1)} = (\tilde{a}_{ij}^{(t+1)})$ is obtained from $A^{(t)}$ according to the formula

$$\tilde{a}_{ij}^{(t+1)} = \begin{cases} a_{ij}^{(t)}, & \text{if } i \leq t + p, \\ a_{ij}^{(t)} - \left(\frac{a_{it}^{(t)}}{a_{i-1,t}^{(t)}}\right)a_{i-1,j}^{(t)}, & \text{if } i \geq t + p + 1 \text{ and } a_{i-1,t}^{(t)} \neq 0, \\ a_{ij}^{(t)}, & \text{if } i \geq t + p + 1 \text{ and } a_{i-1,t}^{(t)} = 0. \end{cases} \quad (1.2)$$

Denote by $E_j(\alpha)$ the elementary matrix obtained from the identity matrix I_n by changing the $(j, j - 1)$ th entry to α . Then the formula (1.2) can be matricially described as follows

$$E_{t+p+1}(-\alpha_{t+p+1,t}) \dots E_n(-\alpha_{nt}) A^{(t)} = \tilde{A}^{(t+1)}$$

which, in return, gives that

Download English Version:

<https://daneshyari.com/en/article/4599317>

Download Persian Version:

<https://daneshyari.com/article/4599317>

[Daneshyari.com](https://daneshyari.com)