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Rank structure properties of rectangular matrices admitting bidiagonal-type factorizations

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ABSTRACT

In this paper, we investigate the class of rectangular matrices that admit bidiagonal-type factorizations by Neville elimination without exchanges. We provide a complete characterization for a rectangular matrix to be factored as a product of bidiagonal factors and a banded factor in terms of rank structure properties. Consequently, we give a complete characterization of the class of rectangular matrices that have Neville factorizations.

1. Introduction

Factorizations of matrices as a product of bidiagonal-type factors have played an important role in many applications. For example, it is well-known that a nonsingular totally nonnegative matrix (i.e., all its minors are nonnegative) can be factored as a product of nonnegative bidiagonal factors [9], and thus, by virtue of the bidiagonal

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factorization, high accurate computations have been successfully performed including computing its inverse, LDU factorization, eigenvalues and SVD [13,14]. The existence and uniqueness of the bidiagonal factorization is critical to these high accurate computations. Because of the importance, much work has been devoted to bidiagonal-type factorizations of matrices and related numerical computations [1,2,4,5,10,12,15]. As has been known, Neville elimination is a very effective tool to derive the bidiagonal-type factorization.

Neville elimination is a classical elimination technique which, differently from Gaussian elimination, produces zeros in a column by subtracting from each row an adequate multiple of the previous one. This elimination method has been precisely described and extensively studied in [6–8,11]. Next let us illustrate the elimination method by reducing a matrix into a banded matrix including a diagonal matrix.

Definition 1.1. Let $A = (a_{ij}) \in \mathbb{C}^{n\times m}$, and let $0 \le p \le n-1$ and $0 \le q \le m-1$. If $a_{ij} = 0$ for all $i - j > p$, then A is a lower p-banded matrix; if $a_{ij} = 0$ for all $j - i > q$, then A is an upper q-banded matrix; if A is lower p-banded and upper q-banded, then A is called a (p, q)-banded matrix.

For a matrix $A = (a_{ij}) \in \mathbb{C}^{n\times m}$, Neville elimination reduces the matrix A into a lower p-banded matrix in the following procedure:

$$A = \tilde{A}^{(1)} \to A^{(1)} \to \tilde{A}^{(2)} \to \ldots \to \tilde{A}^{(r)} \to A^{(r)} \\ \to \tilde{A}^{(r+1)} = A^{(r+1)} = U, \quad r = \min\{n-p-1, m\},$$

where $U \in \mathbb{C}^{n\times m}$ is a lower p-banded matrix, and for all $1 \le t \le r$, $A^{(t)} = (a_{ij}^{(t)})$ is obtained from $\tilde{A}^{(t)}$ by reordering the rows $t + p, \ldots, n$ such that

$$a_{it}^{(t)} = 0, \quad i \ge t+p \quad \Rightarrow \quad a_{ht}^{(t)} = 0, \quad \forall h \ge i; \tag{1.1}$$

$\tilde{A}^{(t+1)} = (\tilde{a}_{ij}^{(t+1)})$ is obtained from $A^{(t)}$ according to the formula

$$\tilde{a}_{ij}^{(t+1)} = \begin{cases} a_{ij}^{(t)}, & \text{if } i \le t+p, \\ a_{ij}^{(t)} - (\frac{a_{it}^{(t)}}{a_{i-1,t}^{(t)}})a_{i-1,j}^{(t)}, & \text{if } i \ge t+p+1 \text{ and } a_{i-1,t}^{(t)} \ne 0, \\ a_{ij}^{(t)}, & \text{if } i \ge t+p+1 \text{ and } a_{i-1,t}^{(t)} = 0. \end{cases} \tag{1.2}$$

Denote by $E_j(\alpha)$ the elementary matrix obtained from the identity matrix I_n by changing the $(j, j-1)$th entry to α. Then the formula (1.2) can be matricially described as follows

$$E_{t+p+1}(-\alpha_{t+p+1,t}) \ldots E_n(-\alpha_{nt})A^{(t)} = \tilde{A}^{(t+1)}$$

which, in return, gives that