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Sparse recovery on Euclidean Jordan algebras

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A B S T R A C T

This paper is concerned with the problem of sparse recovery on Euclidean Jordan algebra (SREJA), which includes the sparse signal recovery problem and the low-rank symmetric matrix recovery problem as special cases. We introduce the notions of restricted isometry property (RIP), null space property (NSP), and s-goodness for linear transformations in s-SREJA, all of which provide sufficient conditions for s-sparse recovery via the nuclear norm minimization on Euclidean Jordan algebra. Moreover, we show that both the s-goodness and the NSP are necessary and sufficient conditions for exact s-sparse recovery via the nuclear norm minimization on Euclidean Jordan algebra. Applying these characteristic properties, we establish the exact and stable recovery results for solving SREJA problems via nuclear norm minimization.

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1. Introduction

The *sparse recovery on Euclidean Jordan algebra* (SREJA) is the problem of recovering a sparse (low-rank) element of a Euclidean Jordan algebra from a number of linear measurements (see Section 2 for more details). The problem of SREJA includes the problems of sparse signal recovery (SSR) and low-rank symmetric matrix recovery (LMR) as special cases. Since $\mathbb{R}^n$ and $\mathbb{S}^n$ (the space of all $n \times n$ real symmetric matrices) are the two simplest Euclidean Jordan algebras (other Euclidean Jordan algebras include, e.g., the Lorenz space $\mathbb{L}^n$ and the Hermitian Space $\mathbb{H}^n$), the problem SREJA is a non-trivial generalization of SSR and LMR. The study of SREJA may disclose more essential properties of the sparse recovery problem, as conjectured by Recht, Fazel, and Parrilo [24], which is the main purpose of this study.

SREJA is generally NP-hard since SSR is a well-known NP-hard problem. In the terminology of compressed sensing (CS), SSR is also called *the cardinality minimization problem, or the l_0-minimization problem*, see the papers by Donoho [9] and Candès, Romberg and Tao [4,6]. In particular, Candès and Tao [6] introduced a restricted isometry property (RIP) of a sensing matrix which guarantees to recover a sparse solution of SSR by ℓ_1-norm minimization. After that, several other sparse recovery conditions were introduced, including the null space properties (NSPs) [8] and the s-goodness [16,17]. The recovery conditions for SSR is important since they open the door for efficient solution of SSR, which has wide applications in signal and image processing, statistics, computer vision, system identification, and control. For more details, see the survey papers [2,23] and a new monograph [10]. Recently, recovery conditions on RIP, NSP and s-goodness for SSR have also been successfully extended to the case of LMR, see, [20, 24–26]. Recht, Fazel, and Parrilo [24] provided a certain RIP condition on the linear transformation of LMR, which guarantees that the minimum nuclear norm solution is a minimum rank solution, they also presented an analysis on the parallels between the cardinality minimization and rank minimization. Recht, Xu, and Hassibi [26] gave the NSP condition for LMR, which is also discussed by Oymak et al. in [22]. Note that the NSP is both necessary and sufficient for exactly recovering a low-rank matrix via nuclear norm minimization problem. Recently, Chandrasekaran, Recht, Parrilo, and Willsky [7] indicated that a fixed s-rank matrix X_0 can be recovered if and only if the null space of $\mathcal{A}$ does not intersect the tangent cone of the nuclear norm ball at X_0. Kong, Tunçel, and Xiu [20] extended the results of [7], studied the concept of s-goodness for the sensing matrix in SSR and the linear transformations in LMR, and established the equivalence of s-goodness and the NSP in the matrix setting.

The study of SREJA is also motivated by the recent development of optimization techniques on Euclidean Jordan algebras, which provide a foundation for solving SREJA via convex relaxation. For details on Euclidean Jordan algebra optimization, see a survey paper [30] and the papers [12,13,19,21,28,29], to name a few. In particular, Recht, Fazel, and Parrilo [24] mentioned the power of Jordan-algebraic approach and asked whether similar results can be obtained in the more general framework of Euclidean Jordan