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On a conjecture about the norm of Lyapunov mappings

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ABSTRACT

The norm properties of Lyapunov mappings and their restrictions on symmetric and skew-symmetric subspaces are investigated. For non-negative, non-positive, and tridiagonal matrices, this paper gives an affirmative answer to an open conjecture which says that the norm of a Lyapunov mapping over the space of real square matrices equals that of its restricted map over the subspace of real symmetric matrices. In addition, matrix transformation and norm relationships on certain matrices arising from the Lyapunov mapping and its restrictions are provided.

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1. Introduction

It is well known that Lyapunov mappings play very important roles in the studying of the stability of linear systems [14]. For a continuous-time linear system $\dot{x}(t) = Ax(t)$ with A being a real square matrix, we have the continuous-time Lyapunov mapping $\mathcal{L}_A : P \mapsto AP + PA^T$. When studying the stability of a system, we try to find $P > 0$ such that the value of the corresponding Lyapunov mapping is negative, that is, to find $P > 0$ satisfying $\mathcal{L}_A(P) < 0$. A matrix A is said to have a quadratic Lyapunov function (QLF) if there exists $P > 0$ such that $\mathcal{L}_A(P) < 0$. It is obvious that A has a QLF if and only if A is Hurwitz, or equivalently, all eigenvalues of A are in the open left-half complex plane.

However, when uncertainty is involved, the robustness of stability is required to be considered. For example, we discuss the stability of linear system $\dot{x}(t) = (A + \Delta A)x(t)$, where A is a known matrix and ΔA is an uncertain term. Hence, one tries to find the conditions on ΔA, which guarantee that $A + \Delta A$ has a common QLF for all permitted ΔA. That is to say, there exists a common $P > 0$ such that $\mathcal{L}_{A+\Delta A}(P) < 0$ for all ΔA. Based on some properties of the Lyapunov mapping, it is proved in [14] that $A + \Delta A$ has a common QLF if $\|\Delta A\| \leq \frac{1}{2}\|\mathcal{L}_A^{-1}\|^{-1}$, see [14, Corollary 1]. Besides this result, some Pythagorean-like conditions are established in [14], which assure the existence of a common QLF for two given matrices. Based on these results, robust stability conditions are derived for a class of linear systems. For a discrete-time switched system $x(k+1) = A_i x(k)$, $i \in \{1, 2, \ldots, m\}$, where A_i is a known matrix, we have m Lyapunov mappings $\mathcal{L}_{A_i} : P \mapsto A_i^T P A_i - P$. When there exists a common $P > 0$ such that $\mathcal{L}_{A_i}(P) < 0$ for all $i \in \{1, 2, \ldots, m\}$, the discrete-time switched system is asymptotically stable under any switching signal. For this switched system, a necessary condition for the existence of a common QLF and a sufficient condition for the non-existence of a common QLF are derived in [18]. Since then, studying the existence of a common QLF to analyze the robust stability of an uncertain system and stability of a switched system has become a hot research area in the control field. For example, it is proved in [4] that a set of block upper triangular matrices share a common QLF, if and only if each set of diagonal blocks shares a common QLF. For two special classes of stable systems, a sufficient condition is derived for the existence of a common QLF, which assures the uniform asymptotic stability of the corresponding switched linear systems [2]. In [17], for $m\,(> 2)$ stable second-order linear systems, some necessary and sufficient conditions are presented for the existence of common QLFs. In [16], the particle swarm optimization method is employed to find a common QLF for a class of switched linear systems. Gradient iteration algorithms for computing a common QLF have been designed in [11] for a large family of stable linear systems. These iterative algorithms converge to a common QLF for a finite number of subsystems and probabilistically converge to a common QLF for infinite ones. In [13], several necessary and sufficient conditions have been presented for the existence of an affine parameter-dependent Lyapunov function, which guarantee the Hurwitz (or Schur) stability of a polytope of matrices. For two stable complex matrices, some necessary and