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An inverse problem for gyroscopic systems ☆

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ABSTRACT

A "gyroscopic system" is a Hermitian matrix-valued function of the form $\mathbb{L}(\lambda) = M\lambda^2 + iG\lambda + C$ where $M, G, C \in \mathbb{R}^{n\times n}$ with $M > 0$ (positive definite), $G^T = -G \neq 0$, $C^T = C$ and may be indefinite. Here we study factorizations of the form $\mathbb{L}(\lambda) = (I\lambda - B)M(I\lambda - A)$, where $A, B \in \mathbb{C}^{n\times n}$, and use them to construct gyroscopic systems with specified right divisor $I\lambda - A$. We examine the constraints on the choice of A.

1. Introduction

We consider quadratic eigenvalue problems (modelling gyroscopic systems) of the form $\mathbb{L}(\lambda)x = 0$ where

$$\mathbb{L}(\lambda) = M\lambda^2 + iG\lambda + C, \tag{1}$$

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and $M, G, C \in \mathbb{R}^{n\times n}$ with $M > 0$, $G^T = -G \neq 0$, $C^T = C$. Since $M > 0$ there is a square-root $M^{1/2} > 0$ in $\mathbb{R}^{n\times n}$ and so, without loss of generality, we may study

$$\mathbb{L}(\lambda) = I\lambda^2 + iG\lambda + C. \tag{2}$$

Notice that, although G and C are real matrices, we have $(iG)^* = iG$ and $C^* = C^T = C$, so we may treat $\mathbb{L}(\lambda)$ as a *Hermitian* matrix polynomial. The reader is referred to [7] for a survey of the theory of such gyroscopic systems. The technique employed here is closely related to that of [8], where a more general class of problems is considered.

It is easily seen that the spectrum of $\mathbb{L}(\lambda)$ has Hamiltonian symmetry. Thus, if λ is an eigenvalue, so is $-\lambda$ and, if λ is not real, $\bar{\lambda}$ and $-\bar{\lambda}$ are also eigenvalues. For example, linear Hamiltonian systems take the form (2) in papers of Chugunova and Pelinovsky [3], and of Kozlov [6]. In contrast with [6] and several earlier studies of gyroscopic systems, C may be indefinite. Earlier contributions of the authors [2] also apply to this problem. However, the context of that work is wider, as the coefficient of λ in (2) is simply a Hermitian matrix.

In this paper we deal with the following inverse problem: Assign a right divisor, $I\lambda - A$ and describe the class of left divisors $I\lambda - B$ for which the quadratic matrix polynomial

$$\mathbb{L}(\lambda) = (I\lambda - B)(I\lambda - A), \quad B, A \in \mathbb{C}^{n\times n}, \tag{3}$$

is a gyroscopic system of the form (2).

The first discussion (Section 2) depends on properties of such pairs of divisors $I\lambda - B$, $I\lambda - A$ as developed in Section 2.5 of [5] – with a history going back to work of H. Langer [9].

In Sections 2.1 and 2.2 the matrix A of (3) is expressed in terms of its real and imaginary parts, $A = A_R + iA_I$ and, assuming that the real part A_R is nonsingular, we prove that B of (3) is uniquely determined in terms of A (see Theorems 4 and 6). However, we cannot expect that, given *any* complex matrix A, there will always be a left divisor B for which $(I\lambda - B)(I\lambda - A)$ is gyroscopic. Necessary conditions on the choice of A are given in Proposition 8 and Theorem 9. Examples are given to illustrate these results.

Given the Hamiltonian symmetry of the spectrum of $\mathbb{L}(\lambda)$, it is necessary for stability that all eigenvalues be **real and semisimple**, i.e. for strong (or marginal) stability, all eigenvalues of $\mathbb{L}(\lambda)$ must be real and semisimple with a definite type (positive or negative). A multiple semisimple real eigenvalue which is not of definite type is said to be of *mixed type*. However, *weak stability* implies that the spectrum is real and semisimple, but there is at least one multiple real eigenvalue with non-unique sign characteristic. A brief introduction to these ideas is included in Appendix A.2.

In Section 3 we assign A to be a Hermitian matrix. Given that A has a nonsingular real part, necessary and sufficient conditions are provided for A to be a right divisor of a gyroscopic system (Theorem 10). Then we take advantage of the semisimple spectrum