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# Further remarks on multivariate polynomial matrix factorizations ☆

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Multivariate ($n$-D) polynomial matrix factorizations are basic research problems in multidimensional systems and signal processing. In this paper Youla's MLP Lemma [19] is extended to the general case. Based on this extension, generalizations of some results in [16] are proved which might be useful for further investigations on problems of factorizations of multivariate polynomial matrices.


## 1. Introduction

The basic structure of multivariate ($n$-D) polynomial matrices and $n$-D polynomial matrix factorizations have been investigated over the past several decades because of their

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wide range of applications in multidimensional ($n$-D) circuits, systems, controls, signal processing, and other related areas (see, e.g., [1–4,7–11,19,20]). Theory and algorithms of univariate matrix factorizations have played an important part in design and analysis of control systems. For the bivariate (2-D) case, polynomial matrix factorizations have been completely solved using the so-called matrix primitive factorizations and Smith reduction over the rational function field of one variable over thirty years ago [14,4].

The factorization problems of $n$-D ($n > 2$) polynomial matrices have been open for almost thirty years. In recent years, some factorization theory and algorithms for a large class of $n$-D polynomial matrices have been developed using a completely different approach from that of the 2-D case [9,10,15–18,12]. Using these new results and algorithms, a new constructive approach for bivariate polynomial matrix factorization was proposed to avoid doing computation over the rational function field in one variable [13].

In [17], some progress about $n$-D factor prime factorization was obtained. Especially, under the condition that every factor is regular, a sufficient and necessary condition was given for factor prime factorizations. Up to now, a complete characterization for factor primeness remains open for an arbitrary $n$-D polynomial matrix, which is a major problem on this research direction. In order to attack this important open problem, some new research methods need to be developed. But for the time being, we hope to check carefully our approach for investigating matrix factorizations. In [16], one of the major theorems is a one-to-one correspondence between existence of an MLP factorization and the freeness of certain submodule. This approach is further extended to general case in [17] using some different proof methods. On the other hand, the methods in [16] might be useful for further research, so this paper attempts to extend the major result in [16] to the general case.

The organization of the paper is as follows. In Section 2, some preliminaries and motivation of the research problem are introduced. In Section 3, we first extend Youla's "MLP Lemma" [19] to the general case, then making use of this result to extend Theorem 4 in [16] to the general case, and an application is presented to explain this result.

## 2. Preliminaries and research problem

Let $n$ be an integer with $n \geq 1$. Let $k$ be an arbitrary but fixed field, $R = k[z_1, \ldots, z_n]$ be the polynomial ring in $n$ variables $z_1, \ldots, z_n$ over $k$. $R^{l\times m}$ denotes the free module of $l \times m$ matrices with entries in $R$. We also write $R^{1\times m}$ as $R^m$ which is a free module of rank $m$ over $R$. Without loss of generality, the size of a given matrix is assumed to be $l\times m$ with $l \leq m$. For computation concerning modules and matrices over the polynomial ring we refer to [5,6].

Let $F \in R^{l\times m}$ be of full row rank, we denote the greatest common divisor of all $i \times i$ minors of $F$ by $\mathrm{d}_i(F)$ for $i = 1, \ldots, l$. As in [16,12], we set $\mathrm{d}(F) = \mathrm{d}_l(F)$ for the g.c.d. of the minors of maximal order of $F$, and $\rho(F)$ denotes the submodule of $R^m$ generated by rows of $F$. Also for $f \in R$, $[fI_l \mid F] \in R^{l\times(l+m)}$ stands for the matrix concatenating $fI_l$ and $F$, where $I_l$ is the identity matrix of order $l$.