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On maximum Estrada indices of bipartite graphs with some given parameters ☆

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The Estrada index of a graph G is defined as $EE(G) = \sum_{i=1}^{n} e^{\lambda_i}$, where $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of the adjacency matrix of G. In this paper, we characterize the unique bipartite graph with maximum Estrada index among bipartite graphs with given matching number and given vertex-connectivity, edge-connectivity, respectively.

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1. Introduction

Let G be a simple graph on n vertices. The eigenvalues of G are the eigenvalues of its adjacency matrix, which are denoted by $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. The Estrada index of G, put forward by Estrada [7], is defined as

$$EE(G) = \sum_{i=1}^{n} e^{\lambda_i}.$$

The Estrada index has multiple applications in a large variety of problems, for example, it has been successfully employed to quantify the degree of folding of long-chain molecules, especially proteins [8–10], and it is a useful tool to measure the centrality of complex (reaction, metabolic, communication, social, etc.) networks [11,12]. There is also a connection between the Estrada index and the extended atomic branching of molecules [13]. Besides these applications, the Estrada index has also been extensively studied in mathematics, see [16,18,20–22]. Ilić and Stevanović [16] obtained the unique tree with minimum Estrada index among the set of trees with a given maximum degree. Zhang, Zhou and Li [20] determined the unique tree with maximum Estrada indices among the set of trees with a given matching number. In [4], Du and Zhou characterized the unique unicyclic graph with maximum Estrada index. Wang et al. [19] determined the unique graph with maximum Estrada index among bicyclic graphs with fixed order, and Zhu et al. [23] determined the unique graph with maximum Estrada index among tricyclic graphs with fixed order. More mathematical properties on the Estrada index can be founded in [14].

A graph is bipartite if its vertex set can be partitioned into two subsets X and Y so that every edge has one end in X and the other end in Y. We denote a bipartite graph G with bipartition (X, Y) by $G[X, Y]$. If $G[X, Y]$ is simple and every vertex in X is joined to every vertex in Y, then G is called a complete bipartite graph. Up to isomorphism, there is a unique complete bipartite graph with parts of sizes m and n, denoted $K_{m,n}$. For an edge subset A of the complement of G, we use $G + A$ to denote the graph obtained from G by adding the edges in A.

A matching in a graph is a set of pairwise nonadjacent edges. If M is a matching, the two ends of each edge of M are said to be matched under M, and each vertex incident with an edge of M is said to be covered by M. A maximum matching is one which covers as many vertices as possible. The number of edges in a maximum matching of a graph G is called the matching number of G and denoted by $\alpha'(G)$. Let $\mathcal{M}_{n,p}$ be the set of bipartite graphs on n vertices with $\alpha'(G) = p$.

A cut vertex (edge) of a graph is a vertex (edge) whose removal increases the number of components of the graph. A (An) vertex (edge) cut of a graph is a set of vertices (edges) whose removal disconnects the graph. The connectivity (edge-connectivity) of a graph G is defined as

$$\kappa(G) = \min\{|S| : S \text{ is a vertex cut of } G\}, \qquad \kappa'(G) = \min\{|S| : S \text{ is an edge cut of } G\}.$$