Linear Algebra and its Applications 465 (2015) 312–324

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Characterizing graphs with maximal Laplacian Estrada index ☆

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ARTICLE INFO

Article history:
Received 1 October 2013
Accepted 20 September 2014
Available online 10 October 2014
Submitted by R. Brualdi

MSC:
05C50
05C35

Keywords:
Estrada index
Laplacian eigenvalues
Laplacian Estrada index
Chromatic number

ABSTRACT

Let G be a simple graph on n vertices. The Laplacian Estrada index of G is defined as $LEE(G) = \sum_{i=1}^{n} e^{\mu_i}$, where $\mu_1, \mu_2, \ldots, \mu_n$ are the Laplacian eigenvalues of G. In this paper, we give some upper bounds for the Laplacian Estrada index of graphs and characterize the connected (n, m)-graphs for $n + 1 \le m \le \frac{3n-5}{2}$ and the graphs of given chromatic number having maximum Laplacian Estrada index, respectively.

☆ Supported by NSFC (No. 11001089), CPSF (Nos. 2012M520815 and 2013T60411).

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http://dx.doi.org/10.1016/j.laa.2014.09.029

1. Introduction

We consider simple graphs. Let G be a graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. For $v \in V(G)$, the degree of v, denoted by $d_G(v)$, is the number of edges incident to v. Let $D(G)$ be the diagonal matrix $\operatorname{diag}(d_G(v_1), d_G(v_2), \ldots, d_G(v_n))$ and $A(G)$ the adjacency matrix of G. The Laplacian matrix of G is $L(G) = D(G) - A(G)$.

The Estrada index of G is defined as [8]

$$EE(G) = \sum_{i=1}^{n} e^{\lambda_i(G)},$$

where $\lambda_1(G), \lambda_2(G), \ldots, \lambda_n(G)$ are the eigenvalues of $A(G)$, arranged in non-increasing order. This graph invariant found various applications and has been studied extensively, see, e.g., [5,7–11,17].

Let $\mu_1(G), \mu_2(G), \ldots, \mu_{n-1}(G), \mu_n(G) = 0$ be the eigenvalues of $L(G)$ arranged in non-increasing order. The Laplacian Estrada index of G is defined as [11]

$$LEE(G) = \sum_{i=1}^{n} e^{\mu_i(G)}.$$

Like the Estrada index, the Laplacian Estrada index is used as a molecular descriptor [3]. Using the power series expansion of the exponential function, we get

$$LEE(G) = \sum_{i=1}^{n} e^{\mu_i(G)} = \sum_{k=0}^{\infty} \frac{M_k(G)}{k!},$$

where $M_k(G) = \sum_{i=1}^{n} \mu_i^k$ is the k-th Laplacian spectral moment of G, which reflects the structural features of networks [20,21] and molecular graphs [3].

Let $\mathcal{L}(G)$ be the line graph of G. For a bipartite graph G with n vertices and m edges, Zhou and Gutman [22] established a relation between $LEE(G)$ and $EE(\mathcal{L}(G))$:

$$LEE(G) = n - m + e^2 \cdot EE\big(\mathcal{L}(G)\big).$$

Bamdad et al. [2] gave a lower bound for Laplacian Estrada index of a graph using the numbers of vertices and edges. Du and Liu [7] determined the unique trees with minimum and maximum Laplacian Estrada indices with some given parameters. Zhou [23] gave lower bounds for Laplacian Estrada index using the degree sequence. More properties of Laplacian Estrada index may be found in, e.g., [2,6,17,18,22–24].

In this paper, we give sharp upper bounds for the Laplacian Estrada index of (n, m)-graphs and characterize the extremal graphs for $n + 1 \le m \le \frac{3n-5}{2}$. We also give a sharp upper bound for the Laplacian Estrada index of a graph using the chromatic number, and characterize the extremal graphs.