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Solvability and feasibility of interval linear equations and inequalities

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ABSTRACT

This paper considers solvability and feasibility of interval linear equations and inequalities. The new concepts of solvability and feasibility are introduced in a unified framework. Some existing concepts such as weak solvability, strong solvability, weak feasibility and strong feasibility are special cases in this framework. Necessary and sufficient conditions for checking new types of solvability and feasibility are developed.

1. Introduction

Uncertainty in data measurement and observation is a common phenomenon in real problem modeling. Considering their interval envelopes is one way to tackle these uncertainties. Systems of interval linear equations and interval linear inequalities frequently arise in situations where the data cannot be measured exactly but are known to be in a certain range. Methods for checking weak and strong solvability and feasibility of interval linear equations and inequalities have been developed e.g. in [1–9].

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In this paper, we will introduce some new concepts of solvability and feasibility of interval linear systems in a unified framework. Weak and strong solvability and feasibility are special cases of this framework. Necessary and sufficient conditions for checking the new types of solvability and feasibility are also developed.

Throughout the paper, we follow the notations given in [6,7]. An interval matrix is defined as

$$\mathbf{A} = [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{m\times n}; \underline{A} \leq A \leq \overline{A}\},$$

where $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, and $\underline{A} \leq \overline{A}$. Similarly, we define an interval vector as one column interval matrix

$$\mathbf{b} = [\underline{b}, \bar{b}] = \{b \in \mathbb{R}^m; \underline{b} \leq b \leq \bar{b}\},$$

where $\underline{b}, \bar{b} \in \mathbb{R}^m$, and $\underline{b} \leq \bar{b}$. The set of all m-by-n interval matrices will be denoted by $\mathbb{IR}^{m\times n}$ and the set of all m-dimensional interval vectors by $\mathbb{IR}^m$.

Denote by A_c and A_Δ the center and radius matrices given by

$$A_c = \frac{1}{2}(\underline{A} + \overline{A}), \qquad A_\Delta = \frac{1}{2}(\overline{A} - \underline{A}),$$

respectively. Then $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$. Similarly, the center and radius vectors are defined as

$$b_c = \frac{1}{2}(\underline{b} + \bar{b}), \qquad b_\Delta = \frac{1}{2}(\bar{b} - \underline{b})$$

respectively. Then $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$.

Let Y_m be the set of all $\{-1, 1\}$ m-dimensional vectors, i.e.

$$Y_m = \{y \in \mathbb{R}^m \mid |y| = e\},$$

where $e = (1, \cdots, 1)^T$ is the m-dimensional vector of all 1's. For a given $y \in Y_m$, let

$$T_y = \operatorname{diag}(y_1, \ldots, y_m)$$

denote the corresponding diagonal matrix. For each $x \in \mathbb{R}^n$, we define its sign vector $\operatorname{sign} x$ by

$$(\operatorname{sign} x)_i = \begin{cases} 1 & \text{if } x_i \geq 0, \\ -1 & \text{if } x_i < 0, \end{cases}$$

where $i = 1, \cdots, n$. Then we have $|x| = T_z x$, where $z = \operatorname{sign} x \in Y_n$.

For a given interval matrix $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta] \in \mathbb{IR}^{m\times n}$, and for each vector $y \in Y_m$ and each vector $z \in Y_n$, we introduce the matrices

$$A_{yz} = A_c - T_y A_\Delta T_z.$$