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Nonlinear maps preserving the minimum and surjectivity moduli ☆

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Let X and Y be infinite-dimensional complex Banach spaces, and let $\mathscr{B}(X)$ (resp. $\mathscr{B}(Y)$) denote the algebra of all bounded linear operators on X (resp. on Y). We describe maps φ from $\mathscr{B}(X)$ onto $\mathscr{B}(Y)$ satisfying

$$\mathrm{c}\big(\varphi(S) \pm \varphi(T)\big) = \mathrm{c}(S \pm T)$$

for all $S, T \in \mathscr{B}(X)$, where $\mathrm{c}(\cdot)$ stands either for the minimum modulus, or the surjectivity modulus, or the maximum modulus. We also obtain analog results for the finite-dimensional case.

1. Introduction

There has been considerable interest in studying nonlinear maps on operators or matrices preserving the invertibility, the spectrum and its parts. In [2], Bhatia, Šemrl

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and Sourour characterized surjective maps preserving the spectral radius of the difference of matrices. In [32], Molnár studied maps preserving the spectrum of operator or matrix products and showed, in particular, that a surjective map φ on the algebra $\mathscr{B}(\mathscr{H})$ of all bounded linear operators on an infinite-dimensional complex Hilbert space $\mathscr{H}$ preserves the spectrum of operator products if and only if there exists an invertible operator $A \in \mathscr{B}(\mathscr{H})$ such that either $\varphi(T) = ATA^{-1}$ for all $T \in \mathscr{B}(\mathscr{H})$ or $\varphi(T) = -ATA^{-1}$ for all $T \in \mathscr{B}(\mathscr{H})$. His results have been extended in several directions for uniform algebras and semisimple commutative Banach algebras, and a number of results is obtained on maps preserving several spectral and local spectral quantities of operator or matrix product, or Jordan product, or Jordan triple product, or difference; see for instance [3–6,11–20,22,23,27–31,33,34,37] and the references therein.

Throughout this paper, X and Y denote infinite-dimensional complex Banach spaces, and $\mathscr{B}(X, Y)$ denotes the space of all bounded linear maps from X into Y. When $X = Y$, we simply write $\mathscr{B}(X)$ instead of $\mathscr{B}(X, X)$. The dual space of X will be denoted by X^*, and the Banach space adjoint of an operator $T \in \mathscr{B}(X)$ will be denoted by T^*. In [19], Havlicek and Šemrl gave a complete characterization of bijective maps φ on the algebra $\mathscr{B}(\mathscr{H})$ of all bounded linear operators on an infinite-dimensional complex Hilbert space $\mathscr{H}$ satisfying the condition (1.1). In [21], Hou and Huang characterized surjective maps between standard operator algebras on complex Banach spaces that completely preserve the spectrum or the invertibility in both directions. They also observed that Havlicek and Šemrl's result and its proof remains valid in the case of Banach spaces setting.

Theorem 1.1. *(See Havlicek and Šemrl [19], Hou and Huang [21].) A map φ from $\mathscr{B}(X)$ onto $\mathscr{B}(Y)$ satisfies*

$$\varphi(S) - \varphi(T) \text{ is invertible} \quad \Longleftrightarrow \quad S - T \text{ is invertible} \tag{1.1}$$

if and only if one of the following situations hold:

(i) *There is an operator $R \in \mathscr{B}(Y)$ and there are bijective continuous mappings $A : X \to Y$ and $B : Y \to X$, either both linear or both conjugate linear, such that*

$$\varphi(T) = ATB + R, \quad T \in \mathscr{B}(X). \tag{1.2}$$

(ii) *There is an operator $R \in \mathscr{B}(Y)$ and there are bijective continuous mappings $A : X^* \to Y$ and $B : Y \to X^*$, either both linear or both conjugate linear, such that*

$$\varphi(T) = AT^*B + R, \quad T \in \mathscr{B}(X). \tag{1.3}$$

This case may occur only if both X and Y are reflexive.

The *minimum modulus* of an operator $T \in \mathscr{B}(X)$ is $\mathrm{m}(T) := \inf\{\|Tx\|: x \in X, \|x\| = 1\}$, and is positive precisely when T is bounded below; i.e., T is injective and has