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A determinantal approach to Sheffer sequences

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In this paper, by using the theory of Riordan arrays and the relations between Sheffer sequences and Riordan arrays, we give a determinantal definition for Sheffer sequences. Based on this new definition, some general properties of Sheffer sequences are reproved, and the determinantal representations of some well-known Sheffer sequences are presented.
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1. Introduction

Sequences of polynomials play a fundamental role in mathematics. One of the most famous classes of polynomial sequences is the class of Sheffer sequences, which contains many important sequences such as those formed by Bernoulli polynomials, Euler polynomials, Abel polynomials, Hermite polynomials, Laguerre polynomials, etc., and contains the classes of associated sequences and Appell sequences as two subclasses. In [16–18],

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Rota, Roman, et al. studied the Sheffer sequences systematically by the theory of modern umbral calculus. In [15], Roman further developed the theory of umbral calculus and generalized the concept of Sheffer sequences so that more special polynomial sequences are included, such as the sequences related to Gegenbauer polynomials, Chebyshev polynomials and Jacobi polynomials.

Many works have been devoted to the study of Sheffer sequences and special Sheffer sequences. For example, in 1999, Costabile [6] proposed a determinantal definition for the classical Bernoulli polynomials, which was used by Costabile, Dell'Accio and Gualtieri [7] to obtain some well-known properties of these polynomials. In 2010, Costabile and Longo [8] further generalized the results in [6,7] and introduced the determinantal definition for Appell sequences. It is interesting to note that Yang also gave the determinantal representation for Appell sequences independently [25]. Yang's result is equivalent to Costabile and Longo's, and was published in 2008.

Since the Bernoulli polynomials form an Appell sequence, and all the Appell sequences are in fact Sheffer sequences, it is natural to extend the determinantal theory to Sheffer sequences. In this paper, we will study this problem by using the relations between Sheffer sequences and Riordan arrays, which were given in [12] for the case of classical Sheffer sequences and classical Riordan arrays and in [9,23] for the case of generalized Sheffer sequences and generalized Riordan arrays.

The paper is organized as follows. Some basic definitions and results related to Sheffer sequences and Riordan arrays are introduced at the end of this section. In Section 2, we give the determinantal definition for Sheffer sequences. In Section 3, by using the determinantal definition and the theory of Riordan arrays, we reprove some general properties of Sheffer sequences, including the conjugate representation, the generating function, the operator characterization, and the umbral composition. Section 4 illustrates the determinantal representations of some well-known Sheffer sequences. Finally, in Section 5, we study briefly two properties of Riordan arrays, which are by-products of our theory.

It should be noted that very recently, another Yang [24] established a different determinantal representation for Sheffer sequences by using an approach based on the production matrices of Riordan arrays.

Now, let us introduce some definitions and results briefly.

Let $\mathbb{K}$ be a field of characteristic zero. Let $\mathscr{F}$ be the set of all formal power series in the variable t over $\mathbb{K}$. Thus an element of $\mathscr{F}$ has the form

$$f(t) = \sum_{k=0}^{\infty} a_k t^k,$$

where $a_k \in \mathbb{K}$ for all $k \in \mathbb{N}$, and $\mathbb{N} := \{0, 1, 2, \ldots\}$. The *order* $o(f(t))$ of a power series $f(t)$ is the smallest integer k for which the coefficient of t^k does not vanish. The series $f(t)$ has a multiplicative inverse, denoted by $f(t)^{-1}$ or $1/f(t)$, if and only if $o(f(t)) = 0$. We call such a series *invertible*. The series $f(t)$ has a compositional inverse, denoted by

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