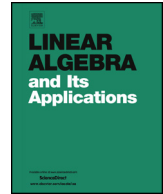




Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa



A modification of eigenvalue localization for stochastic matrices



Chaoqian Li, Yaotang Li^{*}

School of Mathematics and Statistics, Yunnan University, Kunming, Yunnan, 650091, PR China

ARTICLE INFO

Article history:

Received 20 February 2014

Accepted 22 July 2014

Available online 14 August 2014

Submitted by R. Brualdi

MSC:

65F15

15A18

15A51

Keywords:

Stochastic matrix

Eigenvalues

Nonnegative matrices

ABSTRACT

A new eigenvalue localization for stochastic matrices is provided, and is used to estimate the moduli of the subdominant eigenvalue. Numerical examples are given to show that our results are better than those in Cvetković, Kostić and Peña (2011) [3].

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

An entrywise nonnegative matrix $A = [a_{ij}] \in R^{n \times n}$ is called row stochastic (or simply stochastic) if all its row sums are 1, that is,

$$\sum_{j=1}^n a_{ij} = 1, \quad \text{for each } i \in N = \{1, 2, \dots, n\}.$$

^{*} Corresponding author.

E-mail addresses: lichaoqian@ynu.edu.cn (C. Li), liyaotang@ynu.edu.cn (Y. Li).

Obviously, 1 is an eigenvalue of a stochastic matrix with a corresponding eigenvector $e = [1, 1, \dots, 1]^T$. From the Perron–Frobenius Theorem [1], for any eigenvalue λ of A , that is, $\lambda \in \sigma(A)$, we have $|\lambda| \leq 1$, so that in fact 1 is a dominant eigenvalue for A [3]. Here we call λ a subdominant eigenvalue of a stochastic matrix A if $1 > |\lambda| > |\eta|$ for all eigenvalues η different from 1 and λ [3,5,6].

Stochastic matrices and eigenvalue localization of stochastic matrices play key roles in many application fields such as Computer Aided Geometric Design [9], Birth–Death Processes [2,4,7,8], and Markov chain [10].

In [3], L.J. Cvetković et al. presented a region including all eigenvalues of a stochastic matrix A different from 1 by refining the Geršgorin circle [11] of A .

Theorem 1. (See [3, Theorem 3.4].) Let $A = [a_{ij}] \in R^{n \times n}$ be a stochastic matrix, and let s_i be the minimal element among the off-diagonal entries of the i -th column of A , that is, $s_i = \min_{j \neq i} a_{ji}$. Taking $\gamma(A) = \max_{i \in N} (a_{ii} - s_i)$, then for any $\lambda \in \sigma(A) \setminus \{1\}$,

$$|\lambda - \gamma(A)| < r(A) = 1 - \text{trace}(A) + (n - 1)\gamma(A).$$

Although Theorem 1 provides a circle with the center $\gamma(A)$ and radius equal to $1 - \text{trace}(A) + (n - 1)\gamma(A)$ to localize the eigenvalue λ , it is not effective in some cases. Consider the following class of matrices

$$SM_0 = \{A \in R^{n \times n} : A \text{ is stochastic, and } a_{ii} = s_i = 0, \text{ for each } i \in N\}.$$

Then for any $A \in SM_0$, $\text{trace}(A) = 0$ and $\gamma(A) = 0$. Hence by Theorem 1, we have $|\lambda| < 1$ for $\lambda \in \sigma(A) \setminus \{1\}$. This is trivial. It is very interesting how to conquer this drawback.

Note that if A is a stochastic matrix, then A^m is also stochastic for any positive integer m . Therefore, we can apply Theorem 1 to A^m and obtain

$$|\lambda^m - \gamma(A^m)| \leq r(A^m) = 1 - \text{trace}(A^m) + (n - 1)\gamma(A^m). \quad (1)$$

When A is a positive stochastic matrix, L.J. Cvetković et al. [3] proved that the sequence $\{\gamma(A^m)\}$ converges to 0, and the radii of the corresponding circles $r(A^m) = 1 - \text{trace}(A^m) + (n - 1)\gamma(A^m)$ also tend to 0.

Theorem 2. (See [3, Theorem 3.5].) Let $A = [a_{ij}] \in R^{n \times n}$ be a positive stochastic matrix. Then

$$\lim_{m \rightarrow \infty} \gamma(A^m) = 0$$

and

$$\lim_{m \rightarrow \infty} r(A^m) = 0.$$

Download English Version:

<https://daneshyari.com/en/article/4599389>

Download Persian Version:

<https://daneshyari.com/article/4599389>

[Daneshyari.com](https://daneshyari.com)