Journal of Network and Computer Applications 37 (2014) 186–193

Contents lists available at ScienceDirect

Journal of Network and Computer Applications

journal homepage: www.elsevier.com/locate/jnca

# Domatic partition in homogeneous wireless sensor networks

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ARTICLE INFO

*Article history:*
Received 11 April 2012
Received in revised form
6 January 2013
Accepted 18 February 2013
Available online 3 April 2013

*Keywords:*
Wireless sensor networks (WSNs)
Domatic partition (DP)
Domatic number
Clustering
Skyline disk
Congest model

ABSTRACT

In wireless sensor networks, rotating dominating sets periodically is an important technique, for balancing energy consumption of nodes and hence maximizing the lifetime of the networks. This technique can be abstracted as the domatic partition problem, which partitions the set of nodes in networks into disjoint dominating sets. Through rotating each dominating set in the domatic partition periodically, the energy consumption of nodes can be greatly balanced and the lifetime of the network can be prolonged. In order to solve the domatic partition problem, we present a Cell Structure which is constructed as follows. Firstly, the network is divided into clusters, and then a clique is constructed in each cluster. Based on the Cell Structure, we propose a new constant-factor approximation algorithm for domatic partition using the property of the skyline of uniform radius disks. The algorithm is called distributed nucleus algorithm (DNA). In addition, we show that DNA can be implemented in constant rounds in the congest model.



## 1. Introduction

In wireless sensor networks (WSNs), sensor nodes usually are charged with battery whose energy is limited and placed one-time, which makes it impossible for a second charging. Hence, prolonging the lifetime of networks by reducing the energy consumption of nodes is an important challenge. One common method for saving energy is to find a dominating set for data gathering. While there exits a problem that the dominators in the dominating set consume too much extra overheads such as gathering, processing and forwarding data information to consume energy faster than other nodes in the network, which shortens the lifetime of the network. Consequently, it is an important challenge to find a mechanism for balancing energy consumption and prolonging the network lifetime.

Sleep scheduling is a standard approach for balancing energy consumption. By this approach, the nodes make local decisions to sleep or to join the dominating set so that they can take turns being dominators. Thus, each node in the network has a chance to become a dominator and then the energy consumptions among nodes come to balance. The problem of rotating the role of being a dominator has been abstracted as the domatic partition problem. Given a graph $G=(V,E)$ corresponding to a WSN, a dominating set $D \subseteq V$ of $G$ is a subset of vertices such that each node $v \in V$ is either in $D$ or has a neighbor in $D$. A domatic partition (DP) is a partition $\mathcal{D}=\{D_1, D_2, \ldots, D_t\}$ of $V$ such that each $D_i$ is a dominating set of $G$, where $t$ is called domatic number, which is the number of disjoint dominating sets. The domatic partition problem seeks a domatic partition with maximal domatic number.

To understand the motivation, suppose that $\mathcal{D}=\{D_1, D_2, \ldots, D_t\}$ is a DP of $G$. Then a simple sleep scheduling is that the nodes in $D_1$ are activated in a fixed period $T$, during which the rest of the nodes are asleep, followed by the period $T$ in which nodes in $D_2$ are active, while the rest of the nodes are asleep, and so on. After such one-time sleep scheduling, the period of being active of dominators in each dominating set is $T$. The domatic partition has $t$ dominating sets, the period of one-time scheduling is $tT$. Therefore maximizing the domatic number $t$ results in maximizing the period of one-time scheduling, in turn maximizing the lifetime of the network.

Clustering is a basic and effective method of sleep scheduling. It is a fundamental mechanism to design scalable sensor network protocols. A clustering algorithm divides the network into disjoint subsets of nodes such that each subset is a cluster. A good clustering imposes a high-level structure on the network. It is easier to design efficient protocols on this high-level structure than at the level of the individual node. Many efficient clustering protocols (Liu et al., 2007; Zhou et al., 2009; Yu et al., 2011a,b,

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http://dx.doi.org/10.1016/j.jnca.2013.02.025

2012) have been proposed. We propose a uniform distributed clustering (UDC) algorithm, which contributes to find a maximal domatic number.

There are two typical message passing models, that is, local model and congest model (Kuhn et al., 2006), depending on how much information can be sent in each message. The local model is the most fundamental model and it takes no account of the congestion. But in practice, the amount of information exchanged between two neighbors in one communication step is limited. The congest model takes into account the volume of communication and generally limits the information that can be sent in one message to $O(\log n)$ bits. Given the additional restriction, the congest model is significantly stricter than the local model. Our algorithm shows that the domatic partition problem can run constant rounds in the congest model.

The rest of this paper is organized as follows. Section 2 reviews the existing algorithms for the domatic partition problem. Section 3 shows the performance of our algorithm with theoretical analysis. Section 4 gives the results of the simulation. Finally, Section 5 concludes this paper.

## 2. Related work

Graph coloring is a widely used approach to solve the domatic partition problem. The nodes in the graph can be divided into different coloring classes such that each color class is a dominating set of a domatic partition. This approach is commonly used in arbitrary graphs.

In Cardei et al. (2002), in order to achieve energy savings, the authors proposed a centralized algorithm using graph coloring to generate a number of disjoint dominating sets. To maximize the lifetime of a sensor network, Islam et al. (2009) also used the graph coloring to obtain a domatic partition. While both of them fail to give any theoretical analysis about the domatic number bound.

In Feige et al. (2003), applying the graph coloring theory, Feige et al. proved that every graph with maximum degree $\Delta$ and minimum degree $\delta$ contains a domatic partition of size $(1-o(1))(\delta+1)/\ln\Delta$. They turned this proof into a centralized algorithm that produces a domatic partition of $\Omega(\delta/\ln\Delta)$ sets. In Moscibroda and Wattenhofer (2005), the authors defined the domatic partition problem as the maximum cluster-lifetime problem and proposed a randomized, distributed algorithm which is an $O(\log n)-$approximation with high probability in arbitrary graphs. This algorithm is the distributed implementation of the centralized algorithm in Feige et al. (2003). Both of them obtain a logarithmic approximation algorithm for domatic partition.

In Mahjoub and Matula (2010), the authors showed that simple topology-based graph coloring can solve the domatic partition problem in random geometric graphs (RGGs) and provide up to $(\delta+1)$ disjoint $(1-\varepsilon)$ dominating sets on a large range of experimented graphs. Later, they carried the study further in Mahjoub and Matula (2011) by proposing a practical solution to the distributed $(1-\varepsilon)$ dominating sets partition problem that is based on localized graph coloring algorithms. While both of them obtain the approximation only by simulation results rather than theoretical analysis.

The uniform partition is a new approach used in unit disk graphs (UDGs). By this approach, a constant-factor approximation algorithm can be found on UDGs.

Pemmaraju and Pirwani (2006) first proposed the method of the uniform partition and gave three deterministic, distributed algorithms for finding $k$-domatic partition of size at least a constant fraction of the largest possible $(k-1)$-domatic partition for $k>1$. The first algorithm runs in constant time on UDGs assuming that all nodes know their positions in a global coordinate system. The second algorithm runs in $O(\log n)$ time on UDGs dropping knowledge of global coordinates and instead assuming that pairwise distances between neighboring nodes are known. The third algorithm runs in $O(\log(\log n))$ time in growth-bounded graphs dropping all reliance on geometric information and using connectivity information only.

Pandit et al. (2009) first drove a constant-factor distributed algorithm DomPart that can be implemented in $O(\log n)$ rounds of communication in the congest model on UDGs.

Misra and Mandal (2007) proposed a domatic partition based scheme for clusterhead rotation on UDGs from the technical application of the uniform partition idea. While, the clusterhead rotation via re-clustering is a global operation which suffers from significant energy overheads during the rotation. Later, in Misra and Mandal (2009a,b) proposed an efficient rotation scheme using local rotation with the aim of reducing energy consumption in re-clustering. Followed by these works, Misra and Mandal (2009a,b) studied connected domatic partition (CDP) problem, which essentially involves partitioning the nodes $V$ of a graph $G$ into node disjoint connected dominating sets. They developed a distributed algorithm to construct the CDP of size at least $\lfloor(\delta+1)/\beta(c+1)\rfloor-f$, where $\delta$ is the minimum node degree of $G$, $\beta\le 2$, $c\le 11$ is a constant for an UDG, and the expected value of $f$ is $\epsilon\delta|V|$, where $\epsilon\ll 1$ is a positive constant, and $\delta\le 48$.

## 3. Domatic partition using distributed nucleus algorithm (DNA)

In this section, we present a distributed nucleus algorithm (DNA), which includes two steps. Section 3.1 shows the first step with the construction of Cell Structure, which contains the formation of the cluster and clique. Section 3.2 shows the second step with the centralized nucleus algorithm, which finds dominators in each clique to dominate the nodes in each cluster. All the dominators found in each cluster consist of a dominating set of the network. Section 3.3 obtains the distributed implementation of the centralized nucleus algorithm in Section 3.2.

We make some assumptions regarding the network model as follows:

(1) There are $N$ sensor nodes that are distributed at random in a square field with area $\|\mathcal{A}\|$.
(2) All the sensor nodes have the same initial energy and transmission radius $R$. So the network is homogenous and the graph corresponding to the network is an unit disk graph (UDG).
(3) Each sensor node has a globally unique identification *id*.

### 3.1. The construction of Cell Structure

#### 3.1.1. The formation of cluster

This subsection presents a uniform distributed clustering (UDC) algorithm, which uses the degree of nodes as the main parameter of the clusterhead competition. The formation process is divided into three phases: information collection phase with duration $T_1$, clusterhead competition phase with duration $T_2$ and cluster formation phase with duration $T_3$.

*Phase* 1. *Information collection phase*. In this phase, each node broadcasts a $Node_{Msg}$ message with its half sensing radius $R/2$. Meanwhile, each node receives the $Node_{Msg}$ from its neighbor nodes. Thus, each node knows its degree $d$ within the limits of $R/2$. For each node $i$, we give the following formula to calculate its waiting time $t_i$ in the following phase 2:

$$t_i=\begin{cases}\dfrac{D-1}{d}\times\dfrac{T_2}{2}\times\rho & d\ge D\\ \dfrac{T_2}{2}+(1-\dfrac{d}{D-1})\times\dfrac{T_2}{2}\times\rho & d<D\end{cases}\tag{1}$$