

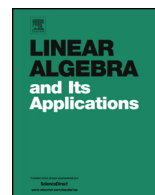


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Local spectrum linear preservers at non-fixed vectors



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ABSTRACT

For a complex $n \times n$ matrix T and a vector $x \in \mathbb{C}^n$, we denote by $\sigma_T(x)$ (respectively, by $r_T(x)$) the local spectrum (respectively, the local spectral radius) of T at x . We prove that $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ linear has the property that for each $T \in \mathcal{M}_n$ there exists a nonzero $x_T \in \mathbb{C}^n$ such that $\sigma_{\varphi(T)}(x_T) = \sigma_T(x_T)$ if, and only if, there exists $A \in \mathcal{M}_n$ invertible such that either $\varphi(T) = ATA^{-1}$ for each $T \in \mathcal{M}_n$, or $\varphi(T) = AT^tA^{-1}$ for each $T \in \mathcal{M}_n$. Modulo a multiplication by a unimodular complex number, we arrive at the same conclusion by supposing that for each $T \in \mathcal{M}_n$ there exists a nonzero $x_T \in \mathbb{C}^n$ such that $r_{\varphi(T)}(x_T) = r_T(x_T)$.

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1. Introduction and statement of the main results

For a fixed integer $n \geq 1$, let us denote by \mathcal{M}_n the space of all $n \times n$ matrices over the complex field \mathbb{C} . For $T \in \mathcal{M}_n$, by $\sigma(T)$ we shall denote its spectrum, that is the set of all eigenvalues of T without counting multiplicities, and by $\rho(T)$ its spectral radius, that is the maximum modulus of $\sigma(T)$. By a result of Marcus and Moyls [12, Theorem 3] and a density argument, if $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ is linear and $\sigma(\varphi(T)) = \sigma(T)$ for each T ,

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then φ is either an automorphism or an anti-automorphism of \mathcal{M}_n : there exists $A \in \mathcal{M}_n$ invertible such that either

$$\varphi(T) = ATA^{-1} \quad (T \in \mathcal{M}_n) \quad \text{or} \quad \varphi(T) = AT^tA^{-1} \quad (T \in \mathcal{M}_n). \tag{1}$$

(Throughout this paper, by T^t we shall denote the transpose of the matrix T .)

Akbari and Aryapoor proved that we arrive at the same conclusion by supposing only that $\sigma(\varphi(T)) \cap \sigma(T) \neq \emptyset$ for each T [1, Corollary 3]. This can be used in order to obtain a new proof for the fact (see [3, Proposition 2]) that a unital linear map $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ satisfying $\rho(\varphi(T)) = \rho(T)$ for each T is also of the form (1). Any such map φ is bijective [13] and therefore preserves the peripheral spectrum [8]. Thus for each T the points $\lambda \in \sigma(\varphi(T))$ for which $|\lambda| = \rho(T)$ are exactly the points $\lambda \in \sigma(T)$ with $|\lambda| = \rho(T)$. Since such a λ exists always, we have that $\sigma(\varphi(T))$ and $\sigma(T)$ have always at least one common element, allowing us to use [1, Corollary 3].

The above results have been generalized in different directions over the last years, and one way to do it is to replace the spectrum function/spectral radius by the local spectrum/local spectral radius at vectors from \mathbb{C}^n . (We refer the reader to the books [11] and [15] for the standard notions from local spectral theory.) For $T \in \mathcal{M}_n$ and $x \in \mathbb{C}^n$, we denote by $\sigma_T(x)$ (respectively $r_T(x)$) the local spectrum (respectively, the local spectral radius) of T at x . Fixing $x_0 \in \mathbb{C}^n \setminus \{0\}$, González and Mbekhta proved in [9] that if $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ is linear and $\sigma_{\varphi(T)}(x_0) = \sigma_T(x_0)$ for every $T \in \mathcal{M}_n$, there exists then an invertible matrix A such that $A(x_0) = x_0$ and $\varphi(T) = ATA^{-1}$ for each $T \in \mathcal{M}_n$. The aim of this paper is to study the same type of linear preserver problem in the case when the vector x_0 is not fixed, but varies with T . A characterization of such maps is given by our first result.

Theorem 1. *Let $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ be a linear map. Then the following statements are equivalent:*

(i) *For each $T \in \mathcal{M}_n$ there exists a nonzero vector $x_T \in \mathbb{C}^n$ such that*

$$\sigma_{\varphi(T)}(x_T) = \sigma_T(x_T). \tag{2}$$

(ii) *For each $T \in \mathcal{M}_n$ there exists a nonzero vector $x_T \in \mathbb{C}^n$ such that*

$$\sigma_{\varphi(T)}(x_T) \cap \sigma_T(x_T) \neq \emptyset. \tag{3}$$

(iii) *The map φ is of the form (1).*

The case of the local spectral radius preservers at some fixed vector $x_0 \in \mathbb{C}^n \setminus \{0\}$ was characterized by Bourhim and Miller in [5]. They proved that if $\varphi : \mathcal{M}_n \rightarrow \mathcal{M}_n$ is linear and $r_{\varphi(T)}(x_0) = r_T(x_0)$ for every $T \in \mathcal{M}_n$, there exist then an invertible matrix A and

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