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Matrix products with constraints on the sliding block relative frequencies of different factors ☆

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ABSTRACT

One of fundamental results of the theory of joint/generalized spectral radius, the Berger–Wang theorem, establishes equality between the joint and generalized spectral radii of a set of matrices. Generalization of this theorem on products of matrices whose factors are applied not arbitrarily but are subjected to some constraints is connected with essential difficulties since known proofs of the Berger–Wang theorem rely on the arbitrariness of appearance of different matrices in the related matrix products. Recently, X. Dai [1] proved an analog of the Berger–Wang theorem for the case when factors in matrix products are formed by some Markov law.
We extend the concepts of joint and generalized spectral radii to products of matrices with constraints on the sliding block relative frequencies of occurrences of different factors, and prove an analog of the Berger–Wang theorem for this case.

1. Introduction

In various theoretic and applied problems there arise the matrix products

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$$M_{\alpha_0} M_{\alpha_1} \cdots M_{\alpha_n}, \quad n \geq 0, \tag{1}$$

where M_i are $(d \times d)$-matrices from a finite collection $\mathcal{M} = \{M_1, M_2, \dots, M_r\}$ with the elements from the field $\mathbb{K} = \mathbb{R}, \mathbb{C}$ of real or complex numbers, and $\boldsymbol{\alpha} = (\alpha_n)$ is a sequence of symbols from the set $\mathcal{A} = \{1, 2, \dots, r\}$, see, e.g., [1–6] and the bibliography therein.

The question about the rate of growth of the matrix products (1) is relatively simple (at least theoretically) in edge cases, for example, when the sequence $\boldsymbol{\alpha} = (\alpha_n)$ is periodic or in (1) all possible sequences $\boldsymbol{\alpha} = (\alpha_n)$ with symbols from $\mathcal{A} = \{1, 2, \dots, r\}$ are considered. In the latter case the question about the rate of growth of all possible matrix products (1) can be answered in terms of the so-called joint or generalized spectral radii of the set of matrices $\mathcal{M}$ [7–10].

In intermediate situations, when the sequences $\boldsymbol{\alpha} = (\alpha_n)$ in (1) are relatively complex but still not totally arbitrary, the question about the rate of growth of the matrix products (1) becomes highly nontrivial and its resolution essentially depends on the structure of the index sequences $\boldsymbol{\alpha} = (\alpha_n)$. In particular, one of the key features of the index sequence in (1) is the frequency p_i of occurrences of the index i.

As a rule, the frequency p_i is defined as the limit of the relative frequencies $p_{i,n}$ of occurrences of the symbol i among the first n members of a sequence. However, one should bear in mind that such a definition of frequency is rather subtle and not very constructive from the point of view of mathematical formalism. Already in the situation when one deals with a single sequence $\boldsymbol{\alpha} = (\alpha_n)$, this definition is not enough informative since it does not answer the question of how often different symbols appear in intermediate, not tending to infinity, finite segments of a sequence. This definition becomes all the less satisfactory in situations when one should deal with not a single sequence but with an infinite collection of such sequences. The principal deficiency here is that the definition of frequency given above does not withstand transition to the limit with respect to different sequences which results in substantial theoretical and conceptual difficulties.

To give “good” properties (from the point of view of ability to use mathematical methods) to determination of frequency one often needs either to require some kind of uniformity of convergence of the relative frequencies $p_{i,n}$ to p_i or to treat appearance of the related symbols in a sequence as a realization of events generated by some random or deterministic ergodic system, and so on. In the latter case, for the analysis of behavior of the matrix products (1), the so-called multiplicative ergodic theorem (in a probabilistic or a measure theoretic setting) is most often used, see, e.g., [11,12]. However, under such an approach one needs to impose rather strong restrictions on the laws of forming the index sequences $\boldsymbol{\alpha} = (\alpha_n)$ which are often difficult to verify of confirm in applications. In all such cases the arising families of the index sequences and of the related matrix products can be rather attractive from the purely mathematical point of view but their description becomes less and less constructive. In applications, it often leads to emergence of an essential conceptual gap or of some kind strained interpretation at use of the related objects and constructions.