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The effect of a graft transformation on distance spectral radius

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We study the effect of a graft transformation on the distance spectral radius of connected graphs. As applications, we determine the unique n-vertex non-starlike tree with maximal distance spectral radius, and the unique n-vertex non-caterpillar trees with minimal and maximal distance spectral radii, and we also consider the distance spectral radius of non-starlike non-caterpillar trees.

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1. Introduction

We consider simple undirected graphs. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. For $u, v \in V(G)$, the distance between vertices u and v, denoted by $d_G(u, v)$ or d_{uv}, is the length of a shortest path connecting them in G.

Let $V(G) = \{v_1, \ldots, v_n\}$. The distance matrix of G is the $n \times n$ matrix $D(G) = (d_{v_i v_j})$. Since $D(G)$ is symmetric, the eigenvalues of $D(G)$ are all real. The distance spectral radius of G, denoted by $\rho(G)$, is the largest eigenvalue of $D(G)$. Since $D(G)$ is irreducible and non-negative, by Perron–Frobenius theorem, there is a unique normalized positive eigenvector of $D(G)$ corresponding to $\rho(G)$, which is called the distance Perron vector of G.

The study of the eigenvalues of the distance matrix began in 1970s by Graham and Pollack [5], Edelberg et al. [3] and Graham and Lovász [4]. Balaban et al. [1] proposed the use of the distance spectral radius as a molecular descriptor, see also [12]. Stevanović and Ilić [8] showed that the n-vertex star S_n is the unique n-vertex tree with minimal distance spectral radius. Ruzieh and Powers [7] and Stevanović and Ilić [8] showed that the n-vertex path P_n is the unique n-vertex connected graph with maximal distance spectral radius. Wang and Zhou [9] determined the unique n-vertex trees with second-minimal, third-minimal, second-maximal and third-maximal distance spectral radii, respectively.

Yu et al. [10] proposed a graft transformation that increases the distance spectral radius under a condition using the components of the distance Perron vector, which (together with some other graft transformations) was then used to determine the graphs with maximal distance spectral radius among connected graphs with fixed numbers of vertices and pendant vertices.

We revisit the graft transformation. We first give a less restricted condition using the components of the distance Perron vector for the graft transformation that increases the distance spectral radius, and then propose some easily checked conditions using only the distances between some vertices for the graft transformation that also increases the distance spectral radius. A main result in [10] and an important tool to study the distance spectral radius established in [8] follow as corollaries.

A tree in which there is at most one vertex of degree at least 3 is said to be starlike. Otherwise, it is non-starlike. A pendant vertex is a vertex of degree 1 in a graph. A caterpillar is a tree such that the deletion of all pendant vertices yields a path. A tree that is not a caterpillar is said to be non-caterpillar. A non-starlike non-caterpillar tree is a tree which is both non-starlike and non-caterpillar.

For $t \geq s \geq 1$, the dumbbell graph $D(n; s, t)$ is the tree obtained by attaching s and t pendant vertices to the two end vertices of the path P_{n-s-t} respectively. Let T be a tree on $n \geq 6$ vertices different from S_n, $D(n; 1, n-3)$ and $D(n; 2, n-4)$. Wang and Zhou [9] showed that

$$\rho(T) > \rho\big(D(n; 2, n-4)\big) > \rho\big(D(n; 1, n-3)\big) > \rho(S_n).$$