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Max algebraic complementary basic matrices

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ABSTRACT

A max algebraic version of the results on complementary basic matrices is presented. It is shown that the max permanent of the result is equal to the product of simpler max permanents and the finite max eigenvalues of the product are the same for any permutation of the basic matrices.

1. Introduction

In a series of papers, [2,4,3], the authors introduced (in [2] the first author) and studied the so called complementary basic matrices, CB-matrices for short. Inspired by the fact

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that in both max algebra and the theory of CB-matrices, cycles play an important role, we intend to build a max algebraic analogy of CB-matrices.

In the most general case of CB-matrices, say of order n, the ordered set of indices $\{1, 2, \dots, n\}$ is split into a union of ordered consecutive intervals $L_1, \dots, L_t$ having the following properties: each L_k has at least two entries; L_1 starts with 1, L_t ends with n; the last number in L_k coincides with the first number of L_{k+1} for all $k = 1, \dots, t-1$. Let us denote by l_k the length of the interval L_k for $k = 1, \dots, t$; thus $l_1 + \dots + l_t$ is equal to $n + t - 1$.

The complementary basic matrices originated in [2] and [3] as follows. Let $C_1, C_2, \dots, C_t$ be square matrices of orders $l_1, l_2, \dots, l_t$ respectively. For $k = 1, \dots, t$, define $n \times n$ matrices G_k by

$$G_k = \begin{bmatrix} I_{p_k} & & \\ & C_k & \\ & & I_{q_k} \end{bmatrix}, \tag{1}$$

where the I's are identity matrices, $p_k = l_1 + \ldots + l_{k-1} - k + 1$, $q_k = n - p_k - l_k$; l_0 is set as zero.

Then for permutations $(i_1, \dots, i_t)$ of $(1, \dots, t)$, products of the form

$$G_{i_1} G_{i_2} \cdots G_{i_t} \tag{2}$$

are called CB-matrices. We shall call them in this way, even though originally CB-matrices were defined only for all l_k equal to 2 and the matrices defined above were called generalized CB-matrices.

It was shown in [4] that for any permutation $(i_1, \dots, i_t)$ of $(1, \dots, t)$, the products $G_{i_1} G_{i_2} \cdots G_{i_t}$ have the same characteristic polynomial and thus the same spectrum. In addition, cycles in C_k generate "big" cycles in the products.

We intend to use the notation and definitions of the max algebra in the sense of those in the recent book of Peter Butkovič [1]. In particular, the basic field is the field $\bar{R}$ of real numbers completed by $-\infty$, denoted simply as ϵ. Max summation, for $a, b \in \bar{R}$, is then denoted as $a \oplus b$ and means $\max(a, b)$ in the usual algebra, max multiplication $a \otimes b$ is then $a + b$ in the usual algebra. Instead of the identity matrices in [4], we have to use the matrices (of the corresponding order)

$$\begin{bmatrix} 0 & \epsilon & \cdots & \epsilon \\ \epsilon & 0 & \cdots & \epsilon \\ . & . & . & . \\ \epsilon & \epsilon & \cdots & 0 \end{bmatrix}, \tag{3}$$

instead of the (block) off-diagonal zeros we have to use again the ϵ's.

The multiplications in (2) will, of course, be by $\otimes$, giving products that we call *max algebraic CB-matrices*, shortly MACB-matrices.