Linear Algebra and its Applications 457 (2014) 428–454

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On classification of 5-dimensional solvable Leibniz algebras

A.Kh. Khudoyberdiyev [a], I.S. Rakhimov [b,*], Sh.K. Said Husain [b]

[a] *Institute of Mathematics, Do'rmon yo'li str. 29, 100125, Tashkent, Uzbekistan*
[b] *Institute for Mathematical Research (INSPEM), Department of Mathematics, FS, Universiti Putra Malaysia, Malaysia*

ARTICLE INFO

Article history:
Received 25 October 2013
Accepted 20 May 2014
Available online 21 June 2014
Submitted by P. Semrl

MSC:
17A32
17A36
17A60
17A65
17B30

Keywords:
Leibniz algebra
Solvability
Nilpotency
Nilradical
Derivation

ABSTRACT

In the paper we describe 5-dimensional solvable Leibniz algebras with three-dimensional nilradical. Since those 5-dimensional solvable Leibniz algebras whose nilradical is three-dimensional Heisenberg algebra have been classified before we focus on the rest cases. The result of the paper together with Heisenberg nilradical case gives complete classification of all 5-dimensional solvable Leibniz algebras with three-dimensional nilradical.

* Corresponding author.
E-mail addresses: khabror@mail.ru (A.Kh. Khudoyberdiyev), risamiddin@gmail.com (I.S. Rakhimov), skartini@science.upm.edu.my (Sh.K. Said Husain).

http://dx.doi.org/10.1016/j.laa.2014.05.034
0024-3795/

1. Introduction

According to the structural theory of Lie algebras a finite-dimensional Lie algebra is written as a semidirect sum of its semisimple subalgebra and the solvable radical (Levi's theorem). The semisimple part is a direct sum of simple Lie algebras which were completely classified in the fifties of the last century. At the same period the essential progress has been made in the solvable part by Mal'cev reducing the problem of classification of solvable Lie algebras to that of nilpotent Lie algebras. Since then all the classification results have been related to the nilpotent part.

Leibniz algebras, a "noncommutative version" of Lie algebras, were first introduced in the mid-1960's by Blokh [4] under the name of D-algebras. They came in again in the 1990's after Loday's work [13], where he introduced calling them Leibniz algebras. During the last 20 years the theory of Leibniz algebras has been actively studied and many results on Lie algebras have been extended to Leibniz algebras (see, e.g. [10,16–18]). Particularly, in 2011 the analogue of Levi's theorem has been proven by D. Barnes [3]. He showed that any finite-dimensional complex Leibniz algebra is decomposed into a semidirect sum of the solvable radical and a semisimple Lie algebra. As above, the semisimple part can be composed by simple Lie algebras and the main issue in the classification problem of finite-dimensional complex Leibniz algebras is to study the solvable part. Therefore the classification of solvable Leibniz algebras is important to construct finite-dimensional Leibniz algebras.

Owing to a result of [14], a new approach for studying the solvable Lie algebras by using their nilradicals was developed [2,6,15,19,20], etc. The analogue of Mubarakzjanov's [14] results has been applied for Leibniz algebras case in [8] which shows the importance of the consideration of their nilradicals in Leibniz algebras case as well. The papers [5, 8,9,11] are also devoted to the study of solvable Leibniz algebras by considering their nilradicals.

The classification, up to isomorphism, of any class of algebras is a fundamental and a very difficult problem. It is one of the first problems that one encounters when trying to understand the structure of a member of this class of algebras. Due to results of [5,7] there are complete lists of isomorphism classes of complex Leibniz algebras in dimensions less then five.

The focus of the present paper is on classification of Leibniz algebras in dimension five. Since the description of the whole of isomorphism classes in 5-dimensional Leibniz algebras seems to be hard we deal with the study of 5-dimensional solvable Leibniz algebras with three-dimensional nilradical. It should be noted that the description of solvable Leibniz algebras with three-dimensional Heisenberg nilradical has been given in [12]. Moreover, it was shown that a 5-dimensional solvable Leibniz algebra with three-dimensional Heisenberg nilradical is a Lie algebra. Therefore, in this paper we don't consider this case.