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# Analytic methods for uniform hypergraphs

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ABSTRACT

This paper presents some analytic methods for studying uniform hypergraphs. Its starting point is the spectral theory of 2-graphs, in particular, the largest and the smallest eigenvalues $\lambda$ and $\lambda_{\min}$ of 2-graphs. First, these two parameters are extended to weighted uniform hypergraphs; second, the eigenvalues-numbers $\lambda$ and $\lambda_{\min}$ are extended to eigenvalues-functions $\lambda^{(p)}$ and $\lambda_{\min}^{(p)}$, which also encompass other graph parameters like the Lagrangian and the number of edges. In this way the functions $\lambda^{(p)}$ and $\lambda_{\min}^{(p)}$ seamlessly join spectral and traditional results in hypergraphs. In particular, this new viewpoint helps to show that spectral extremal and edge extremal problems are asymptotically equivalent.

Naturally, all results about $\lambda^{(p)}$ and $\lambda_{\min}^{(p)}$ also extend spectral hypergraph theory, but delve into deeper problems than before. In fact, the resulting theory is new even for 2-graphs, where some well-settled topics become research challenges again.

The paper covers a multitude of topics, with more than a hundred concrete statements to underpin an analytic theory for hypergraphs. Essential among these topics are a Perron–Frobenius type theory and methods for extremal hypergraph problems.

Many open problems are raised and directions for possible further research are outlined.



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