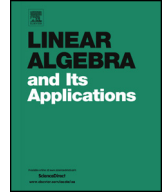




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Linear Algebra and its Applications

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A note on matrices with prescribed off-diagonal submatrix and characteristic polynomial [☆]



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ARTICLE INFO

Article history:

Received 5 September 2013
 Accepted 4 June 2014
 Available online 20 June 2014
 Submitted by R. Brualdi

MSC:
 15A83

Keywords:

Matrix completions
 Inverse eigenvalue problems
 Characteristic polynomial

ABSTRACT

Let $A \in \mathbb{F}^{q \times p}$ be a nonzero matrix over a field \mathbb{F} , and let f be any monic polynomial of degree $n = p+q$ with coefficients in \mathbb{F} . A completion problem proposed and solved by de Oliveira asks for the existence of a matrix of order n with characteristic polynomial f and such that its $q \times p$ submatrix in the left down corner is equal to A . Let $C(f) \in \mathbb{F}^{n \times n}$ be the companion matrix of f . We will construct a matrix $X_A \in \mathbb{F}^{n \times n}$ (which depends exclusively on A) such that $X_A^{-1}C(f)X_A$ is a closed form solution of this completion problem.

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1. Introduction

Let \mathbb{F} be a field and consider the 2×2 block matrix

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad \text{with } M_{11} \in \mathbb{F}^{p \times p} \text{ and } M_{22} \in \mathbb{F}^{q \times q}. \tag{1}$$

[☆] Supported by the Spanish Ministerio de Ciencia y Tecnología grant MTM2009-09180.

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In the seventies de Oliveira posed the problem of determining all possible spectra or characteristic polynomials or invariant polynomials of $[M_{ij}]_{i,j=1}^2$ when some of the blocks are prescribed and the rest of the blocks are free to take any value. Summaries of solved cases can be found in [1,2,4].

In this work we are interested in the particular problem of determining all possible characteristic polynomials of $[M_{ij}]_{i,j=1}^2$ when only M_{21} is prescribed. Note that if M_{21} is the zero matrix then the solution is trivial, namely, a polynomial f of degree $p + q$ is a possible characteristic polynomial if and only if f has a divisor of degree p .

On the other hand, if M_{21} is a nonzero matrix then de Oliveira [3] proved that for any polynomial f of degree $p + q$ there exist some $M_{11} \in \mathbb{F}^{p \times p}$, $M_{12} \in \mathbb{F}^{p \times q}$ and $M_{22} \in \mathbb{F}^{q \times q}$ such that f is the characteristic polynomial of $[M_{ij}]_{i,j=1}^2$. And Ikramov and Chugunov [4] obtained a finite step algorithm to construct concrete M_{11} , M_{12} and M_{22} . In Lemma 1.1 and Theorem 1.1 below we develop a construction that provides a closed form solution. Before we introduce some useful notation.

Definition 1.1. We say that P is a **partially prescribed** matrix over a field \mathbb{F} if some of the entries of P are known elements of \mathbb{F} and the rest of the entries of P are unknown. Each unknown entry of P will be denoted by \square . A **completion** of P is any matrix obtained from P by replacing each unknown entry \square by an element of \mathbb{F} .

For instance, if

$$P = \begin{bmatrix} 3 & \square & 5 \\ \square & \square & 7 \\ 2 & \square & \square \end{bmatrix}, \quad Q = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 1 & 7 \\ 2 & 9 & 6 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 3 & -3 & 5 \\ 1 & 2 & 7 \\ 2 & 8 & 4 \end{bmatrix}$$

then Q and R are two different completions of the partially prescribed matrix P .

Lemma 1.1. Let \mathbb{F} be a field and consider the $(p + q) \times (p + q)$ partially prescribed matrix P over \mathbb{F} which is given by

$$P = \begin{bmatrix} \square & \square \\ A & \square \end{bmatrix} = \left[\begin{array}{ccc|ccc} \square & \cdots & \square & \square & \cdots & \square \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \square & \cdots & \square & \square & \cdots & \square \\ \hline a_{11} & \cdots & a_{1p} & \square & \cdots & \square \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{q1} & \cdots & a_{qp} & \square & \cdots & \square \end{array} \right] \quad \text{where } A \in \mathbb{F}^{q \times p} \text{ and } a_{1p} = 1, \quad (2)$$

and let

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