

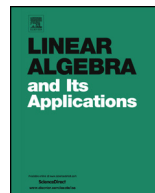


ELSEVIER

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On the term rank partition [☆]Rosário Fernandes ^{b,*}, Henrique F. da Cruz ^a

^b Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal

^a Departamento de Matemática da Universidade da Beira Interior, Rua Marquês D'Ávila e Bolama, 6201-001 Covilhã, Portugal

ARTICLE INFO

Article history:

Received 14 February 2014

Accepted 26 May 2014

Available online 20 June 2014

Submitted by R. Brualdi

MSC:

05A15

05C50

05D15

*Keywords:**t*-Term rank

Interchange

Row sum vector

Column sum vector

ABSTRACT

For each positive integer t , the t -term rank of a $(0, 1)$ -matrix A is the maximum number of 1's in A with at most one 1 in each column and at most t 1's in each row. In [5] R. Brualdi et al. (2012) stated several results for the t -term rank, including a formula for the maximum t -term rank over a nonempty class of $(0, 1)$ -matrices with the same row sum and column sum vectors. In this paper we state more results for the t -term rank. Using these results we define and we study the term rank partition. We also deduce a formula for the minimal t -term rank over a nonempty class of $(0, 1)$ -matrices with the same row sum and column sum vectors.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Let A be an m -by- n , $(0, 1)$ -matrix. Let t be a positive integer. Motivated by the study of combinatorial batch codes (see [4,7,9]), the authors of [5] defined the t -term rank of A ,

[☆] This work was partially supported by *Fundação para a Ciência e Tecnologia* and was done within the activities of the *Centro de Estruturas Lineares e Combinatórias*.

* Corresponding author.

E-mail addresses: mrff@fct.unl.pt (R. Fernandes), hacruz@ubi.pt (H. F. da Cruz).

denoted $\rho_t(A)$, as the maximum number of 1's in A with at most one 1 in each column and at most t 1's in each row.

When $t = 1$, we have the well-known term rank of A , denoted $\rho(A)$. By the König–Egerváry theorem (see [3], p. 6) $\rho(A)$ equals the minimum number of lines that cover all the 1's of A :

$$\rho(A) = \min\{e + f : \exists \text{ a cover of } A \text{ with } e \text{ rows and } f \text{ columns}\}.$$

In [5] the authors established a generalization of this theorem:

Proposition 1.1. *Let A be an m -by- n , $(0, 1)$ -matrix and let t be a positive integer. Then*

$$\rho_t(A) = \min\{te + f : \exists \text{ a cover of } A \text{ with } e \text{ rows and } f \text{ columns}\}. \quad \square$$

Let L be a subset of $\{1, \dots, m\}$ and H be a subset of $\{1, \dots, n\}$. We denote by $A[L; H]$ the $|L|$ -by- $|H|$ submatrix of A determined by the rows with index in L and the columns with index in H . If $L = \{a_1, \dots, a_p\}$ and $H = \{b_1, \dots, b_v\}$ then, sometimes we will write $A[a_1, \dots, a_p; b_1, \dots, b_v]$ instead of $A[L; H]$. When $L = \{1, \dots, m\}$, we write $A[*; H]$ instead of $A[L; H]$.

The t -term rank of A can be obtained using the rank of the matroid union (see [1,5,6,8]) of t copies of $M(A)$, the matroid on $\{1, 2, \dots, n\}$ (n is the number of columns of A), where the independent sets of $M(A)$ are those $K \subseteq \{1, \dots, n\}$ such that, the m -by- $|K|$ submatrix of A , $A[*; K]$, verify $\rho(A[*; K]) = |K|$. Such matroids are transversal matroids and the rank of $M(A)$ is $\rho(A)$. The matroid union of t copies of $M(A)$, this is, the transversal matroid $M(A^{(t)})$ with

$$M(A^{(t)}) = M(A) \vee \dots \vee M(A) \quad (t \text{ copies of } M(A)),$$

has rank equal to $\rho_t(A)$ and its bases (maximal independent sets) are those $K \subseteq \{1, \dots, n\}$ such that $|K| = \rho_t(A[*; K]) = \rho_t(A)$.

Let l be a positive integer. A partition of weight l is a sequence of nonnegative integers $\alpha = (\alpha_1, \dots, \alpha_u)$ satisfying

$$\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_u \geq 0$$

and

$$\sum_{i=1}^u \alpha_i = l.$$

If $\alpha = (\alpha_1, \dots, \alpha_p)$ is a partition of weight l and $\alpha_p > 0$ we say that p is the length of α (that is, the length of α is the greatest k such that $\alpha_k > 0$).

Download English Version:

<https://daneshyari.com/en/article/4599433>

Download Persian Version:

<https://daneshyari.com/article/4599433>

[Daneshyari.com](https://daneshyari.com)