

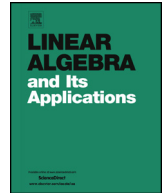


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# Linear Algebra and its Applications

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## Sharp covering of a module by cyclic submodules



I.N. Nakaoka<sup>a,\*,1</sup>, E.L. Monte Carmelo<sup>a,2</sup>,  
O.J.N.T.N. dos Santos<sup>b</sup>

<sup>a</sup> Departamento de Matemática, Universidade Estadual de Maringá, Brazil

<sup>b</sup> Universidade Estadual de Mato Grosso do Sul, Dourados, Brazil

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### ABSTRACT

Let  $A$  be a finite commutative ring with identity. A subset  $H$  of the  $A$ -module  $A^n$  is called an  $R$ -short covering of  $A^n$  if every element of this module can be written as a sum of a multiple of an element in  $H$  and an  $A$ -linear combination with at most  $R$  canonical vectors. Let  $c(A, n, R)$  be the minimum cardinality of an  $R$ -short covering of  $A^n$ . In this work, the numbers  $c(A, n, 0)$  are computed when  $A$  is a direct product of chain rings (extending previous results by Yildiz et al.) and when  $A$  is a finite local ring such that  $D(A)^2 = \{0\}$ , where  $D(A)$  denotes the set of all zero divisors of  $A$ . In order to obtain these results, we develop a method based on action of group, the min–max principle and pairwise weakly linearly independent sets (a concept introduced in this article). A structural connection between classical covering and short covering is described too. Thus a combination of previous results is applied to improve known bounds on short coverings for several values.

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\* Corresponding author.

E-mail addresses: [innakaoka@uem.br](mailto:innakaoka@uem.br) (I.N. Nakaoka), [elmcarmelo@uem.br](mailto:elmcarmelo@uem.br) (E.L. Monte Carmelo), [ojneto@uems.br](mailto:ojneto@uems.br) (O.J.N.T.N. dos Santos).

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## 1. Introduction

### 1.1. Preliminary problems

A collection of subgroups  $\{G_1, \dots, G_k\}$  is a *group partition* of the group  $G$  if  $G$  is the union of these subgroups and each pairwise intersection  $G_i \cap G_j$  contains only the null element of  $G$ . Necessary and sufficient conditions for the existence of a group partition have been investigated since the seminal paper by Miller [11], who proved that if an abelian group has a non-trivial partition, then the group must be an elementary abelian  $p$ -group. Several works about this topic are collected in [19].

An extension arises naturally in the context of vector space. Let  $\mathbb{F}_q^n$  denote the  $n$ -dimensional vector space over the finite field  $\mathbb{F}_q$ . A family  $\{V_i, \dots, V_k\}$  of subspaces of  $\mathbb{F}_q^n$  is called a *vector partition* of  $\mathbb{F}_q^n$  if it satisfies:  $\mathbb{F}_q^n = \bigcup_{i=1}^k V_i$  (covering property) and  $V_i \cap V_j = \{0\}$  for every  $i \neq j$  (packing property). The literature on vector partition interplays theoretical results and applications. Herzog and Schönheim [6] established a connection between vector partitions and single-error-correcting perfect mixed linear codes. Besides coding theory, the partition problem can be applied to several geometric and combinatorial structures: partial spreads, uniformly resolvable designs, see [15] and its references. Let  $\sigma_q(n, t)$  denote the minimum number of a vector partition of  $\mathbb{F}_q^n$  in which the largest partition has dimension  $t$ , see [5]. For our purpose, we mention  $\sigma_q(n, 1) = (q^n - 1)/(q - 1)$ .

### 1.2. Formulation of the problem

This work deals with an extension of vector partition to a module over a finite ring. In view of Miller's result, there is not hope of finding a concept of partition for a general module, because even the principal submodules can have non-trivial intersection. However, we restrict our attention to the corresponding covering problem. Throughout this paper, all rings are assumed to be commutative with identity 1. Let  $A^n$  be the  $A$ -module in the usual way, where  $A$  denotes an arbitrary finite ring.

A subset  $H$  of  $A^n$  is an  *$R$ -short covering* of  $A^n$  if every element in  $A^n$  can be represented as a sum of a multiple of a vector in  $H$  and an  $A$ -linear combination of at most  $R$  canonical vectors of  $A^n$ , that is, for  $v \in A^n$ , there exists  $h \in H$  and there exist scalars  $\alpha, \alpha_1, \dots, \alpha_R \in A$  such that

$$v = \alpha h + \sum_{i=1}^R \alpha_i e_{j_i}, \quad (1.1)$$

where  $\{e_1, \dots, e_n\}$  denotes the canonical base of  $A^n$ . The number  $c(A, n, R)$  represents the minimum cardinality of an  $R$ -short covering of  $A^n$ .

Let us focus on the extremal case  $R = 0$ . A 0-short covering  $H$  of  $A^n$  can be regarded as a *cyclic covering* of  $A^n$ , since  $A^n$  becomes the union of cyclic submodules  $[h] = Ah =$

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