

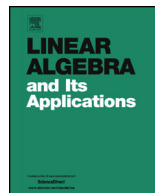


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Resistance distance in subdivision-vertex join and subdivision-edge join of graphs[☆]

Changjiang Bu^{*}, Bo Yan, Xiuqing Zhou, Jiang Zhou

College of Science, Harbin Engineering University, Harbin 150001, PR China

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ABSTRACT

The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting a new vertex into every edge of G . Let $G_1 \cup G_2$ be the disjoint union of two graphs G_1 and G_2 . The subdivision-vertex join of G_1 and G_2 , denoted by $G_1 \vee G_2$, is the graph obtained from $S(G_1) \cup G_2$ by joining every vertex in $V(G_1)$ to every vertex in $V(G_2)$. The subdivision-edge join of G_1 and G_2 , denoted by $G_1 \vee_e G_2$, is the graph obtained from $S(G_1) \cup G_2$ by joining every vertex in $I(G_1)$ to every vertex in $V(G_2)$, where $I(G_1)$ is the set of inserted vertices of $S(G_1)$. In this paper, formulae for resistance distance in $G_1 \vee G_2$ and $G_1 \vee_e G_2$ are obtained.

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1. Introduction

All graphs considered in this paper are simple and undirected. The resistance distance is a distance function on graphs introduced by Klein and Randić [7]. For two vertices u, v in a connected G , the *resistance distance* between u and v is defined to be the effective resistance between them when unit resistors are placed on every edge of G . Let $r_{uv}(G)$

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^{*} Corresponding author.

E-mail addresses: buchangjiang@hrbeu.edu.cn (C. Bu), zhoujiang04113112@163.com (J. Zhou).

denote the resistance distance between u and v in G . Some results on resistance distance can be found in [2,3,5–7,9–11].

For a graph G , let A_G and B_G denote the adjacency matrix and vertex-edge incidence matrix of G , respectively. The matrix $L_G = D_G - A_G$ is called the *Laplacian matrix* of G , where D_G is the diagonal matrix of vertex degrees of G . For a matrix M , the $\{1\}$ -inverse of M is a matrix X such that $MXM = M$. If M is singular, then it has infinite $\{1\}$ -inverses. It is known that resistance distances in a connected graph G can be obtained from any $\{1\}$ -inverse of L_G (see [1,2]).

The subdivision graph $\mathcal{S}(G)$ of a graph G is the graph obtained by inserting a new vertex into every edge of G . Let $G_1 \cup G_2$ be the disjoint union of two graphs G_1 and G_2 . The *subdivision-vertex join* of G_1 and G_2 , denoted by $G_1 \dot{\vee} G_2$, is the graph obtained from $\mathcal{S}(G_1) \cup G_2$ by joining every vertex of $V(G_1)$ to every vertex of $V(G_2)$. The *subdivision-edge join* of G_1 and G_2 , denoted by $G_1 \dot{\vee} G_2$, is the graph obtained from $\mathcal{S}(G_1) \cup G_2$ by joining every vertex of $I(G_1)$ to every vertex of $V(G_2)$, where $I(G_1)$ is the set of inserted vertices of $\mathcal{S}(G_1)$ (see [8]). In this paper, formulae for resistance distance in $G_1 \dot{\vee} G_2$ and $G_1 \dot{\vee} G_2$ are obtained.

2. Preliminaries

For a square matrix M , the *group inverse* of M , denoted by $M^\#$, is the unique matrix X such that $MXM = M$, $XXM = X$ and $MX = XM$. It is known that $M^\#$ exists if and only if $\text{rank}(M) = \text{rank}(M^2)$. If M is real symmetric, then $M^\#$ exists and $M^\#$ is a symmetric $\{1\}$ -inverse of M . Actually, $M^\#$ is equal to the Moore–Penrose inverse of M since M is symmetric (see [4]).

We use $M^{(1)}$ to denote any $\{1\}$ -inverse of a matrix M . Let $(M)_{uv}$ denote the (u, v) -entry of M .

Lemma 2.1. (See [1,4].) *Let G be a connected graph. Then*

$$r_{uv}(G) = (L_G^{(1)})_{uu} + (L_G^{(1)})_{vv} - (L_G^{(1)})_{uv} - (L_G^{(1)})_{vu} = (L_G^\#)_{uu} + (L_G^\#)_{vv} - 2(L_G^\#)_{uv}.$$

Let e denote the all-ones column vector.

Lemma 2.2. *Let S be a real symmetric matrix such that $Se = 0$. Then $S^\#e = 0$, $e^\top S^\# = 0$.*

Proof. Since $Se = 0$, we have $S^\#e = S^\#SS^\#e = (S^\#)^2Se = 0$, $e^\top S^\# = (S^\#e)^\top = 0$. \square

Lemma 2.3. *Let $L = \begin{pmatrix} L_1 & L_2 \\ L_2^\top & L_3 \end{pmatrix}$ be the Laplacian matrix of a connected graph. If each column vector of L_2^\top is $-e$ or a zero vector, then $N = \begin{pmatrix} L_1^{-1} & 0 \\ 0 & S^\# \end{pmatrix}$ is a symmetric $\{1\}$ -inverse of L , where $S = L_3 - L_2^\top L_1^{-1} L_2$.*

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