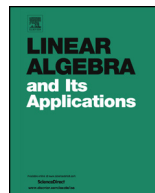




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Lie n -derivations of unital algebras with idempotents

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ABSTRACT

Let \mathcal{A} be a unital algebra with nontrivial idempotents. We show that under certain assumptions every Lie n -derivation φ on \mathcal{A} is of the form $\varphi = d + \delta + \gamma$, where d is a derivation of \mathcal{A} , δ is both a singular Jordan derivation and an antiderivation of \mathcal{A} , and γ is a linear map from \mathcal{A} to its center $Z(\mathcal{A})$ that vanishes on $[\cdots [[\mathcal{A}, \mathcal{A}], \mathcal{A}], \dots, \mathcal{A}]$. As an application we give a description of Lie n -derivations of unital algebras with wide idempotents.

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1. Introduction

Throughout the paper, by an algebra we shall mean an algebra over a fixed unital commutative ring R , and we assume without further mentioning that $\frac{1}{2} \in R$.

Let \mathcal{A} be an algebra. Recall that a linear map $d : \mathcal{A} \rightarrow \mathcal{A}$ is called a *derivation* if $d(xy) = d(x)y + xd(y)$ for all $x, y \in \mathcal{A}$. Similarly, a linear map $d : \mathcal{A} \rightarrow \mathcal{A}$ is called an *antiderivation* if $d(xy) = d(y)x + yd(x)$ for all $x, y \in \mathcal{A}$. Set $[x, y] = xy - yx$ and $x \circ y = xy + yx$ for $x, y \in \mathcal{A}$. A linear map $d : \mathcal{A} \rightarrow \mathcal{A}$ is called *Lie derivation* if

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$d([x, y]) = [d(x), y] + [x, d(y)]$ for all $x, y \in \mathcal{A}$. Similarly, a linear map $d : \mathcal{A} \rightarrow \mathcal{A}$ is called *Jordan derivation* if $d(x \circ y) = d(x) \circ y + x \circ d(y)$ for all $x, y \in \mathcal{A}$. Set $p_1(x) = x$ and

$$p_n(x_1, x_2, \dots, x_n) = [p_{n-1}(x_1, x_2, \dots, x_{n-1}), x_n]$$

for an integer $n \geq 2$. Thus, $p_2(x_1, x_2) = [x_1, x_2]$, $p_3(x_1, x_2, x_3) = [[x_1, x_2], x_3]$, etc. More generally, a linear map $\varphi : \mathcal{A} \rightarrow \mathcal{A}$ is called a *Lie n -derivation* if

$$\varphi(p_n(x_1, x_2, \dots, x_n)) = \sum_{i=1}^n p_n(x_1, \dots, x_{i-1}, \varphi(x_i), x_{i+1}, \dots, x_n)$$

for all $x_1, x_2, \dots, x_n \in \mathcal{A}$. Lie n -derivations were introduced by Abdullaev [1], where the form of Lie n -derivations of a certain von Neumann algebra was described. A Lie 3-derivation is said to be a *Lie triple derivation* (see [5] for details).

Lie (triple) derivations on several classes of algebras have been investigated by many authors (see [2,3,7,9–14,16,17]). In most cases it turns out that a Lie (triple) derivation $\Delta : \mathcal{A} \rightarrow \mathcal{A}$ is of the so-called standard form, i.e., the sum $\Delta = d + \gamma$, where $d : \mathcal{A} \rightarrow \mathcal{A}$ is a derivation and $\gamma : \mathcal{A} \rightarrow Z(\mathcal{A})$ is a central linear map. In 2012, Benkovič and Eremita [5] gave a sufficient condition for a Lie n -derivation on triangular algebras to be of the standard form, which has been generalized to generalized matrix algebras in [18].

We now assume that \mathcal{A} is an algebra with an idempotent $e \neq 0, 1$ and let f denote the idempotent $1 - e$. In this case \mathcal{A} can be represented in the so-called Peirce decomposition form

$$\mathcal{A} = e\mathcal{A}e + e\mathcal{A}f + f\mathcal{A}e + f\mathcal{A}f,$$

where $e\mathcal{A}e$ and $f\mathcal{A}f$ are subalgebras with unitary e and f , respectively, $e\mathcal{A}f$ is an $(e\mathcal{A}e, f\mathcal{A}f)$ -bimodule and $f\mathcal{A}e$ is an $(f\mathcal{A}f, e\mathcal{A}e)$ -bimodule. We will assume that \mathcal{A} satisfies

$$\begin{aligned} exe \cdot e\mathcal{A}f = \{0\} &= f\mathcal{A}e \cdot exe \quad \text{implies} \quad exe = 0, \\ e\mathcal{A}f \cdot fxf = \{0\} &= fxf \cdot f\mathcal{A}e \quad \text{implies} \quad fxf = 0 \end{aligned} \quad (1)$$

for all $x \in \mathcal{A}$. The property (1) was introduced by Benkovič and Širovnik [6]. Some specific examples of unital algebras with nontrivial idempotents having the property (1) are triangular algebras, matrix algebras, and prime algebras with nontrivial idempotents (see [4,6] for details). We remark that generalized matrix algebras can be viewed as special unital algebras with nontrivial idempotents having the property (1) (see [19] for details).

We say that a Jordan derivation $\delta : \mathcal{A} \rightarrow \mathcal{A}$ is a *singular Jordan derivation* according to the decomposition $\mathcal{A} = e\mathcal{A}e + e\mathcal{A}f + f\mathcal{A}e + f\mathcal{A}f$, if

$$\delta(e\mathcal{A}e + f\mathcal{A}f) = 0, \quad \delta(e\mathcal{A}f) \subseteq f\mathcal{A}e, \quad \delta(f\mathcal{A}e) \subseteq e\mathcal{A}f.$$

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