Linear Algebra and its Applications 458 (2014) 605–625

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

A limit formula for joint spectral radius with p-radius of probability distributions

M. Ogura *, C.F. Martin

Department of Mathematics and Statistics, Texas Tech University, Broadway and Boston, Lubbock, TX 79409-1042, USA

ARTICLE INFO

Article history:
Received 13 January 2014
Accepted 19 June 2014
Available online 10 July 2014
Submitted by T. Damm

MSC:
15A60
15A48
93D05
93E15

Keywords:
Joint spectral radius
p-Radius
Lyapunov functions
Absolute exponential stability

ABSTRACT

In this paper we show a characterization of the joint spectral radius of a set of matrices as the limit of the p-radius of an associated probability distribution when p tends to ∞. Allowing the set to have infinitely many matrices, the obtained formula extends the results in the literature. Based on the formula, we then present a novel characterization of the stability of switched linear systems for an arbitrary switching signal via the existence of stochastic Lyapunov functions of any higher degrees. Numerical examples are presented to illustrate the results.

1. Introduction

The joint spectral radius of a set of matrices, originally introduced in the short note [1], is a natural extension of the spectral radius of a single matrix and has found various applications in, for example, wavelet theory, functional analysis, and systems and control

* Corresponding author.
E-mail addresses: masaki.ogura@ttu.edu (M. Ogura), clyde.f.martin@ttu.edu (C.F. Martin).

http://dx.doi.org/10.1016/j.laa.2014.06.034
0024-3795/

theory (see the monograph [2] for detail). This wide range of applications has motivated many authors to study the computation of joint spectral radius. Though even the approximation of joint spectral radius is in general an NP-hard problem [3], there are now a vast amount of efficient methods for the approximation of joint spectral radius [4–6] and also their implementations on mathematical softwares [7].

The result [4] by Blondel and Nesterov is of a particular theoretical interest because it characterizes joint spectral radius as the limit of another joint spectral characteristics called L^p-norm joint spectral radius when p tends to ∞. Given a finite set $\mathcal{M} = \{A_1, \dots, A_N\}$ of real and square matrices of a fixed dimension and a parameter $p \geq 1$, the L^p*-norm joint spectral radius* (p*-radius* for short) of $\mathcal{M}$ is defined by

$$\rho_{p,\mathcal{M}} := \lim_{k\to\infty}\left(N^{-k}\sum_{i_1,\dots,i_k\in\{1,\dots,N\}}\|A_{i_k}\cdots A_{i_1}\|^p\right)^{1/kp}, \tag{1}$$

where $\|\cdot\|$ denotes any matrix norm. Firstly introduced [8,9] for $p = 1$ and then extended [10] for a general p, L^p-norm joint spectral radius has found many applications in various areas of applied mathematics (see [11] and references therein). In particular p-radius has an application to the stability theory of a stochastic switched system [12–14], which is a dynamical system whose structure randomly experiences abrupt changes [15, 16].

Recently this "original" version of L^p-norm joint spectral radius was extended to probability distributions [13]. Roughly speaking, the extension makes it possible to consider the p-radius of a set of *infinitely* many matrices and is useful when, for example, one wants to study the stability of a stochastic switched system with infinitely many subsystems that naturally arise as a result of uncertainty in modeling of dynamical systems. Being an extension, the p-radius of distributions inherits [13] from the p-radius of sets of matrices the characterization [10] as the spectral radius of a matrix. Though the characterization is valid only either when p is an even integer or when matrices in $\mathcal{M}$ leave a common proper cone invariant, it still covers several interesting cases that appear in the stability analysis of stochastic switched linear systems. Then it is natural to expect that the other properties of the p-radius of sets of matrices can be extended to the p-radius of distributions.

In this paper we show that the characterization by Blondel and Nesterov [4] is still valid when we use the p-radius of probability distributions. This extension in particular circumvents the finiteness limitation of the original characterization. Since the proof for the original result relies on the finiteness of the number of matrices, it cannot be directly applied to the current setting. Instead, our proof extensively utilizes so-called cone linear absolute norms [17] and the approximation of a given set of possibly infinitely many matrices by subsets having a certain uniformity property.

As a theoretical application of the characterization of joint spectral radius, we will discuss the stability of switched linear systems. We will present a novel characteriza-