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Generic rank-one perturbations of structured regular matrix pencils

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ABSTRACT

Classes of regular, structured matrix pencils are examined with respect to their spectral behavior under a certain type of structure-preserving rank-1 perturbations. The observed effects are as follows: On the one hand, generically the largest Jordan block at each eigenvalue gets destroyed or becomes of size one under a rank-1 perturbation, depending on that eigenvalue occurring in the perturbating pencil or not. On the other hand, certain Jordan blocks of T-alternating matrix pencils occur in pairs, so that in some cases, the largest block cannot just be destroyed or shrunk to size one without violating the pairing. Thus, the largest remaining Jordan block will typically increase in size by one in these cases. Finally, these results are shown to carry over to the classes of palindromic and symmetric matrix pencils.

1. Introduction

Low-rank perturbations of matrices have been studied by different authors in [9,19–22]. It is well-understood that under a rank-1 perturbation generically (i.e., under a typical

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perturbation) the largest Jordan block of a matrix corresponding to each eigenvalue is destroyed. Generalizations to matrix pencils have been made in recent years; it was shown in [3] that if a regular pencil is subjected to a low-rank perturbation, at each eigenvalue generically not only certain Jordan blocks will disappear but also certain Jordan blocks will become of size one. On the other hand, the behavior of singular pencils under low-rank perturbations was examined in [2] and shown to be very different: generically, existing regular blocks are preserved and certain singular blocks become regular ones.

Structure-preserving rank-1 perturbations were investigated in [16–18] for different types of structured matrices. The focus of [16] lay on J-Hamiltonian matrices, which are known to show a certain pairing of blocks in Jordan canonical form, leading to restrictions on the Jordan form of the perturbed matrix since it is required to be J-Hamiltonian as well. Resulting from this, in some cases the generic behavior was observed to include one block growing in size by one so that the Jordan structure of the perturbed matrix has the pairing characteristic for J-Hamiltonian matrices; this effect is substantially different from the unstructured case. A similar pattern could be identified for real H-skew-symmetric matrices under real H-nonnegative rank-1 perturbations investigated in [4].

In this work, we want to examine structure-preserving rank-1 perturbations of the following classes of structured matrix pencils.

Definition 1.1. A matrix pencil $\lambda E - A$ with $E, A \in \mathbb{C}^{n,n}$ is called:

- *T-even* if E is skew-symmetric and A is symmetric.
- *T-odd* if E is symmetric and A is skew-symmetric.
- *T-alternating* if it is either T-even or T-odd.
- *T-palindromic* if $E = -A^T$.
- *T-anti-palindromic* if $E = A^T$.
- *palindromic* if it is either T-palindromic or T-anti-palindromic.
- *symmetric* if E and A are both symmetric.

Our motivation for considering these classes of structured matrix pencils is that they frequently occur in various applications. A T-palindromic matrix pencil is, e.g., obtained from the vibration analysis of rail tracks under periodic excitation. As described in [8], this problem is modeled by an eigenvalue problem of the form

$$\frac{1}{\kappa}\left(A_0^T + \kappa A_1 + \kappa^2 A_0\right)y = 0,$$

where $A_1, A_0 \in \mathbb{C}^{n,n}$ and $A_1 = A_1^T$. Now, a matrix polynomial, i.e., an expression of the form $P(\lambda) = \sum_{j=0}^{k} \lambda^j A_j$, where $A_j \in \mathbb{C}^{n,n}$ for $j = 0, \dots, n$, is called T-palindromic if $P(\lambda)^T = \lambda^k P(1/\lambda)$ holds. Observe that this definition is consistent with the above definition of T-palindromic matrix pencils and that $A_0^T + \lambda A_1 + \lambda^2 A_0$ is indeed T-palindromic. Such polynomial eigenvalue problems are usually treated by linearization, i.e., by solv-