Linear Algebra and its Applications 459 (2014) 1–12

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On an open problem concerning regular magic squares of odd order ☆

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A R T I C L E   I N F O

Article history:
Received 21 March 2014
Accepted 28 June 2014
Available online 10 July 2014
Submitted by V. Mehrmann

MSC:
15A15
15A18
15B05
15B36

Keywords:
Magic squares
Regular magic squares
Circulant matrices
Fourier matrices
Eigenvalues

A B S T R A C T

In this paper we give an answer to an open problem posed by M.Z. Lee et al. (2012) [2]. More precisely, we prove that the classical regular magic square of odd order produced by the centroskew $\mathcal{S}$-circulant matrix with the assignment $a_j = j-1$, $j = 1, 2, \cdots, (n+1)/2$ is always nonsingular. Moreover an explicit formula for computing the eigenvalues of classical regular magic squares of odd order is given.

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☆ This research is supported by National Natural Science Foundation of China, No. 11171220.

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http://dx.doi.org/10.1016/j.laa.2014.06.046

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1. Introduction

A *magic square* of order n is a square matrix with entries such that the sum of the elements along each row and column, as well as the main diagonal and main backdiagonal are the same constant, the constant is called *magic sum*. In addition, if the entries of magic squares are integers from 1 through n^2, where each number is used exactly once, such magic squares shall be called *classical magic squares*. For a classical magic square of order n, it is well known that the magic sum $\mu = n(n^2+1)/2$. A magic square $M = (m_{ij})_{n\times n}$ is said to be regular if $m_{ij} + m_{n+1-i,n+1-j} = 2\mu/n$.

Magic squares not only have a sense of beauty but also contain many strange mysteries [1]. Along with the rapid development of computer technology, magic squares are widely applied in mathematics and computer science, especially in image processing, graph theory, cryptography. Therefore, more and more scholars draw considerable attention to magic squares [2–6].

M.Z. Lee et al. [2] provided a method of constructing regular magic squares of order n using the centroskew $\mathcal{S}$-circulant matrix A, it was proved that if n is an odd prime power, and the first row of A satisfy $a_j = j-1$ for $1 \leqslant j \leqslant (n+1)/2$, then the classical regular magic square $M = nA + AJ + \frac{n^2+1}{2}E$ is always nonsingular. Meanwhile the following open problem was posed in [2].

Problem. Suppose that A is a centroskew $\mathcal{S}$-circulant matrix of order n, and the first row of A is defined as $a_j = j-1$, $j = 1, 2, \cdots, (n+1)/2$. Whether the classical regular magic square

$$M = nA + AJ + \frac{n^2+1}{2}E$$

is nonsingular when n is a product involving two or more distinct primes.

In this research we prove that the regular magic square defined as above is nonsingular for any odd integer n ($n \geqslant 3$). This is an answer to the above open problem. In addition an explicit formula of eigenvalues is given.

2. Preliminaries

In this section we introduce some basic concepts and symbols which are needed throughout this paper.

Let I_n stand for the identity matrix of order n. For a matrix A, $A(i,j)$ represents the (i,j)-entry of A and A^* is the conjugate transpose of A.

Definition 2.1. (See [7].) A matrix A of order n with the first row $(a_1, a_2, \cdots, a_n)$ is called *circulant matrix*, if the i-th row of A is obtained from the $(i-1)$-th row by shifting the entries cyclically one column to right ($2 \leqslant i \leqslant n$). We shall denote $A = \operatorname{circ}(a_1, a_2, \cdots, a_n)$, i.e.

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