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Bicyclic oriented graphs with the second largest skew-energy ☆

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ABSTRACT

Let G^σ be an oriented graph and $P_s(G^\sigma; x) = \det(xI - S(G^\sigma))$ be the skew-characteristic polynomial of its skew-adjacency matrix $S(G^\sigma)$. The skew-energy of G^σ is defined to be the sum of the absolute values of eigenvalues of $S(G^\sigma)$. In this paper, we firstly find a novel relation between the coefficients of skew-characteristic polynomial of unicyclic graph with the third largest skew-energy and bicyclic oriented graphs, and then use it along with other techniques to characterize the bicyclic oriented graphs with the second largest skew-energy.

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1. Introduction

In theoretical chemistry, the energy of a given molecular graph is related to the total π-electron energy of the molecule represented by that graph. Consequently, the graph energy has some specific chemistry interests and has been extensively studied, since the concept of the energy of simple undirected graphs was introduced by Gutman in [7]. For details we refer the reader to the survey [8] and the book [12].

Recently, the *skew-energy*, introduced by Adiga et al. [1], drew many researchers' attention. Let G be a simple undirected graph with vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$ and edge set $E(G)$, and let G^σ be an oriented graph of G with the orientation σ, which assigns to each edge of G a direction so that the induced graph G^σ becomes an oriented graph or oriented graph. The *skew-adjacency matrix* of G^σ is the $n \times n$ matrix $S(G^\sigma) = [s_{ij}]$, where $s_{ij} = 1$ and $s_{ji} = -1$ if (v_i, v_j) is an arc of G^σ; otherwise $s_{ij} = s_{ji} = 0$. Then the *skew-energy* $\mathcal{E}_S(G^\sigma)$ is defined as the sum of the absolute values of all the eigenvalues of $S(G^\sigma)$. The characteristic polynomial $\det(xI - S(G^\sigma))$ of the skew adjacency matrix $S(G^\sigma)$ of an oriented graph G^σ is called the *skew-characteristic polynomial* of G^σ, written as $P_S(G^\sigma; x) = \sum_{k=0}^{n} b_k(G^\sigma)x^{n-2k}$. Since $S(G^\sigma)$ is a real skew symmetric matrix, we have $b_{2k}(G^\sigma) \geq 0$ and $b_{2k+1}(G^\sigma) = 0$ for all $0 \leq i \leq \lfloor \frac{n}{2} \rfloor$ (see [6]). Then we have

$$P_S\left(G^\sigma; x\right) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} b_{2k}\left(G^\sigma\right)x^{n-2k}.$$

By the coefficients of $P_S(G^\sigma; x)$, the skew energy $\mathcal{E}_S(G^\sigma)$ can be expressed by the following integral formula as follows [11]:

$$\mathcal{E}_S\left(G^\sigma\right) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{1}{t^2} \ln\left(1 + \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} b_{2k}t^{2k}\right) \mathrm{d}t.$$

Thus $\mathcal{E}_S(G^\sigma)$ is a strictly monotonically increasing function of $b_{2k}(G^\sigma)$, $k = 0, 1, \ldots, \lfloor \frac{n}{2} \rfloor$. Consequently, if G^{σ_1} and H^{σ_2} are oriented graphs with

$$b_{2k}\left(G^{\sigma_1}\right) \geq b_{2k}\left(H^{\sigma_2}\right) \quad \text{for each } k \tag{1}$$

for each k $(0 \leq k \leq \lfloor \frac{n}{2} \rfloor)$, then

$$\mathcal{E}_S\left(G^\sigma\right) \geq \mathcal{E}_S\left(H^\sigma\right). \tag{2}$$

Equality in (2) is attained only if (1) is an equality for all $0 \leq k \leq \lfloor \frac{n}{2} \rfloor$. If the inequalities (1) hold for all k, then we write $G \succeq H$ or $H \preceq G$. If $G \succeq H$, but not $H \succeq G$, then we write $G \succ H$. That is exactly the *quasi-order relation* defined by Gutman and Polansky [9] on graph energy, which is generalized to the skew-energy of oriented graph.

Let P_n and C_n be respectively the *path* and the *cycle* of order n. The disjoint union of graphs G and H is denoted by $G \cup H$, and kG stands for the disjoint union of k copies of G. If W be a subset of $V(G)$ and $\overline{W} = V(G^\sigma)\backslash W$, then the orientation $\overrightarrow{G'}$ of G

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