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Some results on B-matrices and doubly B-matrices [☆,☆☆]

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ABSTRACT

A real matrix with positive row sums and all its off-diagonal elements bounded above by their corresponding row means was called in [4] a B-matrix. In [5], the class of doubly B-matrices was introduced as a generalization of the previous class. We present several characterizations and properties of these matrices and for the class of B-matrices we consider corresponding questions for subdirect sums of two matrices (a general 'sum' of matrices introduced in [1] by S.M. Fallat and C.R. Johnson, of which the direct sum and ordinary sum are special cases), for the Hadamard product of two matrices and for the Kronecker product and sum of two matrices.

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1. Introduction

A square real matrix $A = (a_{ij})_{i,j=1}^n$ with positive row sums is a *B-matrix* if all its off-diagonal elements are bounded above by the corresponding row means (see [4]), that is, for all $i \in \{1, \ldots, n\}$,

$$\sum_{j=1}^{n} a_{ij} > 0$$

and

$$\frac{1}{n}\sum_{j=1}^{n} a_{ij} > a_{ik}, \quad \forall k \neq i.$$

In [2] it was proved that these matrices have positive determinants and the author provided a first application to the localization of the real eigenvalues of a real matrix. In [4] the author proved that the class of B-matrices is a subset of the class of P-matrices and applied this property to the localization of the real parts of all eigenvalues of a real matrix.

Given a real matrix $A = (a_{ij})$ we define, for each row i, $r_{i_A} = \max\{0, a_{ij} \mid j \neq i\}$. We simply refer to r_i if the context is unambiguous. If A is a square matrix of order n, let A^+ be the following matrix

$$A^+ = \begin{bmatrix} a_{11} - r_1 & a_{12} - r_1 & \ldots & a_{1n} - r_1 \\ a_{21} - r_2 & a_{22} - r_2 & \ldots & a_{2n} - r_2 \\ \vdots & \vdots & & \vdots \\ a_{n1} - r_n & a_{n2} - r_n & \ldots & a_{nn} - r_n \end{bmatrix}.$$

Throughout this paper, $\mathcal{Z}_n$ will stand for the set of real square matrices of order n whose off-diagonal entries are nonpositive, that is $\mathcal{Z}_n = \{A = (a_{ij}) \in \mathcal{M}_n(\mathbb{R}) \ : \ a_{ij} \leq 0$ if $i \neq j, \ i, j = 1, \ldots, n\}$. If A is in $\mathcal{Z}_n$, we say that A is a Z-matrix (of order n).

In [4] Peña derived a characterization of B-matrices using the values r_i: he proved that a real matrix $A = (a_{ij})_{i,j=1}^n$ is a B-matrix if and only if, for all $i \in \{1, ..., n\}$,

$$\sum_{k=1}^{n} a_{ik} > nr_i. \tag{1}$$

He also proved (see [4]) that A is a B-matrix if and only if, for all $i \in \{1, ..., n\}$,

$$(a_{ii} - r_i) > \sum_{k \neq i} (r_i - a_{ik}).$$