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On linear preservers of g-tridiagonal majorization on $\mathbb{R}^n$

Ali Armandnejad [a,*], Saeedeh Mohtashami [a], Mina Jamshidi [b]

[a] *Department of Mathematics, Vali-e-Asr University of Rafsanjan, P.O. Box 7713936417, Rafsanjan, Iran*

[b] *Graduate University of Advanced Technology, Kerman, Iran*

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ABSTRACT

For vectors $x, y \in \mathbb{R}^n$, it is said that x is g-tridiagonal majorized by y (written as $x \prec_{gt} y$) if there exists a tridiagonal g-doubly stochastic matrix D such that $x = Dy$. In this paper, we continue the previous work (Armandnejad and Gashool (2012) [3]) and give a complete characterization of linear operators preserving $\prec_{gt}$ on $\mathbb{R}^n$.

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1. Introduction

In the recent years, the concept of majorization has been attended specially and the linear operators preserving majorization have attracted much attention from some researchers in linear algebra. Assume that $\mathbb{R}^n$ (respectively $\mathbb{R}_n$) is the vector space of all real $n \times 1$ (respectively $1 \times n$) vectors. Let $\sim$ be a relation on $\mathbb{R}^n$. A linear operator

* Corresponding author.
E-mail addresses: armandnejad@mail.vru.ac.ir (A. Armandnejad), saeede.mohtasham@iran.ir (S. Mohtashami), m.jamshidi@kgut.ac.ir (M. Jamshidi).

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$T : \mathbb{R}^n \to \mathbb{R}^n$ is said to be a linear preserver of $\sim$ if for all $x, y \in \mathbb{R}^n$, $Tx \sim Ty$ whenever $x \sim y$. If T is a linear preserver of $\sim$ and $Tx \sim Ty$ implies that $x \sim y$, then T is called a strong linear preserver of $\sim$. An $n \times n$ nonnegative matrix D is called doubly stochastic if all its row and column sums equal one. For $x, y \in \mathbb{R}^n$, it is said that x is vector majorized by y (written as $x \prec y$) if there exists a doubly stochastic matrix D such that $x = Dy$. In [1], Ando characterized all linear preservers of $\prec$ on $\mathbb{R}^n$. In fact he proved the following proposition. The letters $\mathbf{J}$ and $\mathbf{e}$ are used for the matrix and the vector with all entries equal one respectively (the sizes of $\mathbf{J}$ and $\mathbf{e}$ are understood from the content).

Proposition 1.1. *Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear preserver of $\prec$. Then T has one of the following forms:*

(i) $Tx = \operatorname{tr}(x)a$ for some $a \in \mathbb{R}^n$.
(ii) $Tx = \alpha\Pi x + \beta\mathbf{J}x$ for some $\alpha, \beta \in \mathbb{R}$ and permutation Π.

Some other kinds of majorization with their linear preservers were introduced in [2–8].

Definition 1.2. Consider the affine function $\mathcal{A} : \mathbb{R}^{n-1} \to \mathbf{M}_n$ with

$$\mathcal{A}_\mu := \mathcal{A}(\mu) = \begin{pmatrix} 1-\mu_1 & \mu_1 & & & 0 \\ \mu_1 & 1-\mu_1-\mu_2 & \mu_2 & & \\ & & & \ddots & \mu_{n-1} \\ 0 & & & \mu_{n-1} & 1-\mu_{n-1} \end{pmatrix},$$

where $\mu = (\mu_1, \ldots, \mu_{n-1})^t \in \mathbb{R}^{n-1}$. Every element of $\Omega_n^t := \operatorname{Im}(\mathcal{A})$ is called a tridiagonal g-doubly stochastic matrix.

Definition 1.3. Let $x, y \in \mathbb{R}^n$. We say that x is g-tridiagonally majorized by y (written as $x \prec_{gt} y$) if there exists a tridiagonal g-doubly stochastic matrix $D \in \Omega_n^t$ such that $x = Dy$.

In [3], the authors found the structure of strong linear preservers of $\prec_{gt}$ on $\mathbb{R}^n$ as follows:

Proposition 1.4. *Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator. Then T strongly preserves $\prec_{gt}$ if and only if there exist $\alpha, \beta \in \mathbb{R}$ such that $\alpha(\alpha + n\beta) \neq 0$ and one of the following holds:*

(i) $Tx = \alpha x + \beta\mathbf{J}x$, $\forall x \in \mathbb{R}^n$.
(ii) $Tx = \alpha Px + \beta\mathbf{J}x$, $\forall x \in \mathbb{R}^n$,

where P is the backward identity matrix.

In this paper we characterize the linear operator preserving $\prec_{gt}$ on $\mathbb{R}^n$.

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