

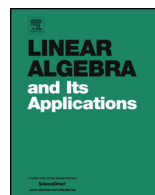


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The principal rank characteristic sequence over various fields



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ABSTRACT

Given an $n \times n$ matrix, its principal rank characteristic sequence is a sequence of length $n + 1$ of 0s and 1s where, for $k = 0, 1, \dots, n$, a 1 in the k th position indicates the existence of a principal submatrix of rank k and a 0 indicates the absence of such a submatrix. The principal rank characteristic sequences for symmetric matrices over various fields are investigated, with all such attainable sequences determined for all n over any field with characteristic 2. A complete list of attainable sequences for real symmetric matrices of order 7 is reported.

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Minor
 Rank
 Symmetric matrix
 Hermitian matrix
 Finite field

1. Introduction

Given an $n \times n$ symmetric matrix A over some field \mathbb{F} , the *principal rank characteristic sequence* of A (abbreviated pr-sequence or $\text{pr}(A)$) is defined as $\text{pr}(A) = r_0]r_1r_2 \cdots r_n$ where

$$r_k = \begin{cases} 1 & \text{if } A \text{ has a principal submatrix of rank } k; \\ 0 & \text{otherwise.} \end{cases}$$

Note that $r_0 = 1$ if and only if A has a 0 diagonal entry. Brualdi et al. [1] introduced the definition of a pr-sequence for a real symmetric matrix as a simplification of the principal minor assignment problem as stated in [4]; see also [6]. In [1] there is also mention of the case $\mathbb{F} = \mathbb{C}$ and the complex Hermitian matrix case. Note that here we denote a pr-sequence by $r_0]r_1r_2 \cdots r_n$ (rather than by $r_0r_1r_2 \cdots r_n$ as in [1]) to visually emphasize the special nature of r_0 .

We use the following result to determine the rank, and hence to work with pr-sequences. Here $A[S|T]$ denotes the submatrix of A in rows indexed by the set S and columns indexed by the set T . If $S = T$, then we write $A[S]$ for the principal submatrix lying in rows and columns indexed by the set S .

Theorem 1.1. *If $A \in \mathbb{F}^{n \times n}$ is symmetric, or $A \in \mathbb{C}^{n \times n}$ is Hermitian, then $\text{rank } A = \max\{|S| : \det(A[S]) \neq 0\}$ (where the maximum over the empty set is defined to be 0).*

Proof. This is immediate from [3, Corollary 8.9.2] for symmetric matrices, and for $A \in \mathbb{C}^{n \times n}$ Hermitian it follows from the equality of algebraic and geometric multiplicity of the eigenvalue zero. \square

All matrices in this paper are square, and unless specified otherwise all matrices are symmetric. We are interested in which pr-sequences are *attainable*, i.e., can be attained by some matrix, and also which sequences are *forbidden*, i.e., no matrix attains the sequence. The case $\mathbb{F} = \mathbb{R}$ was studied by Brualdi et al. [1], and in this paper we continue the investigation into pr-sequences by considering the problem over different fields (Sections 2 and 3) and extending the results of [1] over \mathbb{R} (Section 4). In particular, in Section 3 we identify all attainable pr-sequences of all orders over any field with characteristic 2. For some results we use the $(0, 1)$ adjacency matrix of a graph G , denoted by $A(G)$, and in Section 5 we give results for pr-sequences of such matrices with full rank.

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