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Contractive maps on operator ideals and norm inequalities

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ABSTRACT

Let $(\mathcal{I}, |||.|||)$ be a norm ideal of operators equipped with a unitarily invariant norm $|||.|||$. We employ a technique introduced by K.H. Neeb, and used later by H. Kosaki and G. Larotonda to prove that certain ratios of linear operators acting on operators in $\mathcal{I}$ are contractive. This leads to new inequalities which are sharper than those proved by F. Kittaneh, and by L. Zou and C. He. We also lift a variety of inequalities to the operator setting which were proved in the matrix setting earlier.
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1. Introduction

Let $\mathbb{B}(\mathscr{H})$ denote the algebra of all bounded linear operators on a complex separable Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle)$. The cone of positive operators is denoted by $\mathbb{B}(\mathscr{H})_+$. We shall consider a norm ideal $(\mathcal{I}, |||.|||)$ of $\mathbb{B}(\mathscr{H})$ equipped with a unitarily invariant norm and for notational convenience we shall denote throughout this by $\mathcal{I}$ instead of $(\mathcal{I}, |||.|||)$. As usual, we shall denote the operator norm by $\|\cdot\|$ and by L_X, R_Y the left, right

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multiplication maps on $\mathbb{B}(\mathscr{H})$, respectively, i.e. $L_X(T) = XT$ and $R_Y(T) = TY$. Since L_X and R_Y commute, we have

$$e^{L_X+R_Y}(T) = e^X T e^Y.$$

Let X be selfadjoint member of $\mathbb{B}(\mathscr{H})$ and let $A \in \mathbb{B}(\mathscr{H})$ be arbitrary. Then unitary invariance of the operator norm leads to the elementary inequality

$$\|A \pm iXA\| \geq \|A\|.$$

The above inequality may be written as

$$\big\|(I \pm iL_X)A\big\| \geq \|A\|. \tag{1.1}$$

Let $A, B \in \mathbb{B}(\mathscr{H})_+$ and $X \in \mathbb{B}(\mathscr{H})$. The well known Heinz inequality says that the function,

$$f(\alpha) = \big\|A^\alpha XB^{1-\alpha} + A^{1-\alpha}XB^\alpha\big\|$$

is monotonically decreasing for $\alpha \in [0, 1/2]$.

In 1993, Bhatia and Davis [4] obtained following generalizations of the Heinz inequality:

$$2\big|\big|\big|A^{1/2}XB^{1/2}\big|\big|\big| \leq \big|\big|\big|A^\alpha XB^{1-\alpha} + A^{1-\alpha}XB^\alpha\big|\big|\big| \leq |||AX + XB|||, \quad (0 \leq \alpha \leq 1).$$

More precisely, it was proved that $f(\alpha)$ is a convex function on $[0, 1]$.

A difference version of the Heinz inequality,

$$\big|\big|\big|A^\alpha XB^{1-\alpha} - A^{1-\alpha}XB^\alpha\big|\big|\big| \leq |2\alpha - 1|\,|||AX - XB|||, \quad (0 \leq \alpha \leq 1) \tag{1.2}$$

was proved by Bhatia and Davis [5] in 1995. For more such inequalities the reader may see [16].

A further generalization, namely,

$$\frac{2+t}{2}\big|\big|\big|A^\alpha XB^{1-\alpha} + A^{1-\alpha}XB^\alpha\big|\big|\big| \leq \big|\big|\big|AX + XB + tA^{1/2}XB^{1/2}\big|\big|\big|$$

was proved for $t \in (-2, 2]$, and $\alpha \in [1/4, 3/4]$, see Zhan [22] and Singh, Aujla and Vasudeva [20].

In 2010, Kittaneh [15], gave other generalizations of the Heinz inequality using convexity and the Hermite–Hadamard integral inequality for $0 \leq \mu \leq 1$:

$$\begin{aligned} 2\big|\big|\big|A^{1/2}XB^{1/2}\big|\big|\big| &\leq \frac{1}{|1-2\mu|}\left|\int_\mu^{1-\mu} \big|\big|\big|A^\nu XB^{1-\nu} + A^{1-\nu}XB^\nu\big|\big|\big|\,d\nu\right| \\ &\leq \big|\big|\big|A^\mu XB^{1-\mu} + A^{1-\mu}XB^\mu\big|\big|\big| \end{aligned} \tag{1.3}$$

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