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Construction and minimality of coordinated linear systems

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ABSTRACT

Coordinated linear systems are a particular class of hierarchical systems with a top-to-bottom information structure, consisting of a coordinator system and two or more subsystems. This paper deals with the construction and minimality of coordinated linear systems. Construction procedures are given to transform unstructured or interconnected systems into coordinated linear systems, using the geometric (i.e. basis-independent) concepts of observability and controllability subspaces. Several concepts of minimality for coordinated linear systems are suggested and characterized in order to identify decompositions which are 'as decentralized as possible'. The extension of the developed methods and results to the broader class of hierarchical linear systems is discussed.

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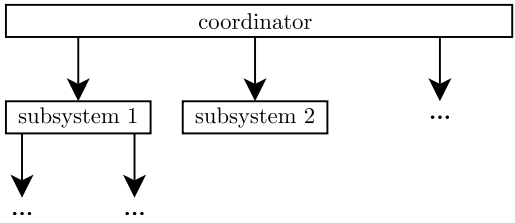

Fig. 1. Scheme of a coordinated system.

1. Introduction

The class of decentralized dynamical systems considered in this paper is that of coordinated linear systems, a special class of hierarchical systems. Coordinated linear systems are structured linear systems consisting of one coordinator system and two or more subsystems, each with their own input and output. The coordinator subsystem may influence the other subsystems but is not influenced by them, and when disregarding the influence of the coordinator, the subsystems are independent. This corresponds to a hierarchical system with two layers and a top-to-bottom information structure, as illustrated in Fig. 1.

Coordinated linear systems were first introduced and characterized in [15], and first results concerning their construction and minimality properties were presented in [6]. A study of the controllability and observability properties of coordinated linear systems can be found in [8], and two different approaches to linear-quadratic (LQ) optimal control for this class of systems are presented in [10,9].

Possible applications of coordinated linear systems arise when several subsystems require interaction (i.e. coordination) to meet a joint control objective. This may apply to linear systems with an inherent hierarchical structure, but also to other types of interconnected systems, which permit a hierarchical modeling approach.

Inherently hierarchical systems include traffic networks and power networks, with the major roads or power lines at a higher level than the side streets or local distribution lines. Other examples are groups of autonomous vehicles with a leader-follower structure, such as vehicle platoons and formations (see [7]): Platoons are typically modeled by chain structures, with the first vehicle at the highest level, and in formations the first vehicle may have several direct followers.

Other interconnected systems can be transformed into coordinated systems, where the coordinator consists of those parts of each system that are relevant to the other systems, and the subsystems consist of the remaining parts of each system. This corresponds to imposing a hierarchy on the different parts of a decentralized system, in order to facilitate decentralized control synthesis. Moreover, large-scale monolithic systems can be decomposed into subsystems with a hierarchical information structure in order to reduce the computational effort needed for control synthesis.

This paper develops an in-depth mathematical analysis of the construction and minimality of coordinated linear systems, focusing on the following questions: