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Corrigendum

Corrigendum to "Jordan homomorphisms of upper triangular matrix rings" [Linear Algebra Appl. 439 (12) (2013) 4063–4069]

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ABSTRACT

In this paper we will present a new proof of the main theorem in Linear Algebra Appl. 439 (2013) 4063-4069.

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Since φ is not a Jordan isomorphism, we see that the statement "Note that $M_1 \cap M_2 = 0$ as $e'\mathcal{T}_{n'}(R)f' \cap f'\mathcal{T}_{n'}(R)e' = 0$" is false (see [1, p. 4066, line 8]). Therefore, the proof of [1, Theorem 2.1] is incorrect. We now present a new proof of [1, Theorem 2.1] as follows:

Proof. We may assume that $\varphi(e_{11})$ is a nontrivial idempotent in $\mathcal{T}_{n'}(R)$ (see [1, p. 4065, lines 8–34]).

Set $A = R$, $M = R^{n-1}$, and $B = \mathcal{T}_{n-1}(R)$. Then $\mathcal{T}_n(R)$ can be viewed as the triangular ring

$$\begin{pmatrix} A & M \\ & B \end{pmatrix}.$$

Set $e = e_{11}$ and $f = \sum_{i=2}^{n} e_{ii}$. Note that e and f are the units of A and B, respectively. Set $e' = \varphi(e)$ and $f' = \varphi(f)$. Then both e' and f' are nontrivial idempotents in $\mathcal{T}_{n'}(R)$ such that $e' + f' = 1_{\mathcal{T}_{n'}(R)}$ (see [1, p. 4065, lines 35–39]). Thus

$$\mathcal{T}_{n'}(R) = e'\mathcal{T}_{n'}(R)e' + e'\mathcal{T}_{n'}(R)f' + f'\mathcal{T}_{n'}(R)e' + f'\mathcal{T}_{n'}(R)f'.$$

Moreover, $\varphi(A) = e'\mathcal{T}_{n'}(R)e'$, $\varphi(B) = f'\mathcal{T}_{n'}(R)f'$, and $\varphi(M) = e'\mathcal{T}_{n'}(R)f' + f'\mathcal{T}_{n'}(R)e'$ as φ is surjective (see [1, p. 4066, lines 1–6]).

Set $M_1 = \varphi^{-1}(e'\mathcal{T}_{n'}(R)f') \cap M$ and $M_2 = \varphi^{-1}(f'\mathcal{T}_{n'}(R)e') \cap M$. We claim that $\varphi(M_1) = e'\mathcal{T}_{n'}(R)f'$ and $\varphi(M_2) = f'\mathcal{T}_{n'}(R)e'$. For every $a + b + m \in \varphi^{-1}(e'\mathcal{T}_{n'}(R)f')$, where $a \in A$, $b \in B$, and $m \in M$, we get that

$$\varphi(a) + \varphi(b) + \varphi(m) \in e'\mathcal{T}_{n'}(R)f'.$$

This implies $\varphi(a) = 0$, $\varphi(b) = 0$, and $\varphi(m) \in e'\mathcal{T}_{n'}(R)f'$. It follows that $m \in M_1$ and

$$\varphi(m) = \varphi(a + b + m).$$

This implies that $\varphi(M_1) = e'\mathcal{T}_{n'}(R)f'$. Similarly, we get that $\varphi(M_2) = f'\mathcal{T}_{n'}(R)e'$.

We next claim that $M = M_1 + M_2$. For every $m \in M$, we see that

$$\varphi(m) \in e'\mathcal{T}_{n'}(R)f' + f'\mathcal{T}_{n'}(R)e' = \varphi(M_1) + \varphi(M_2).$$

Then there exist $m_1 \in M_1$, $m_2 \in M_2$ such that

$$\varphi(m) = \varphi(m_1) + \varphi(m_2).$$

Set $m_0 = m - m_1 - m_2$. It is clear that $\varphi(m_0) = 0$ and then $m_0 \in M_1 \cap M_2$. Write $m = (m_0 + m_1) + m_2$, where $m_0 + m_1 \in M_1$, $m_2 \in M_2$. We see that $M = M_1 + M_2$.