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Semi-Markov migration process in a stochastic market in credit risk

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ABSTRACT

In the present the idea of stochastic Market environment comes into play to express the changes in general economy, which affects any industry in small or great amounts of turbulence. We model the evolution of the Market among its possible ν-states as an $\mathcal{F}$-inhomogeneous semi-Markov process. This idea leads us to modeling the migration process of defaultable bonds as ν different $\mathcal{F}$-inhomogeneous semi-Markov process. The survival probabilities of a defaultable bond in every credit grade are found. The asymptotic behaviour of the survival probabilities is established under certain conditions. Also, it is proved under what conditions the convergence is geometrically fast. The stochastic foundation of the general stochastic discrete-time Market is provided, by proving that the market is viable, if and only if, there exists an equivalent martingale measure, from which we construct the forward probability measure and under which the discounted default free bond price process for all possible states of the Market is a martingale. The term structure of credit spread and the change of real-world probability measure to forward probability measure are studied. In the form of a theorem it is proved that under certain conditions, changing the real probability measure to a forward probability measure, does not affect the inhomogeneous semi Markov process modeling the migration of defaultable bonds. That is, it is proved that it only changes the basic parameters and we provide a relation among the transition probabilities under the two measures. Finally, parameter estimation and calibration of

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the inhomogeneous semi-Markov chain in stochastic environment is being provided.

1. Introductory notes

In the last decade, rating models in credit risk management have become very popular. These systems use the rating of a company or a state as the decisive variable and not – like the formerly used structural models the value of the firm – when it comes to evaluate the default risk of a bond or loan. The popularity is due to straightforwardness of the approach but also to the new Capital Accord (Basel II) of the Basel Committee on Banking Supervision (2001), a regulatory body under the Bank of International Settlements (BIS). Basel II allows banks to base their capital requirements on internal as well as external rating systems. The modeling of the migration process is an important issue for risk management and pricing. Systematic changes in migration matrices have substantial effects on credit Value-at-Risk (VaR) of a portfolio but also on prices of credit derivatives like Collateralized Debt Obligations (CDOs). Therefore, rating transition matrices are of particular interest for determining the economic capital figures like expected loss and VaR for credit portfolios, but can also be helpful as it comes to the pricing of more complex products in the credit industry.

The modeling of the evolution of credit migration started as a Markov chain either in discrete or continuous time by Jarrow and Turnbull [22], Jarrow, Lando and Turnbull [23], and Kijima [26]. Bielecki and Rutkowski [3] reviewed these models and the models proposed by other researchers, who based their work on the simple Markov chain, and worked out their mathematical foundation their common core and interrelations. In the study by Carty and Fons [5] drawn from Moody's Investor service proprietary database, with data span from 1976 to 1993, it was established, that the duration of stay in a specific credit rating class followed the Weibull distribution. The estimation of the parameters of the particular Weibull distribution varied for each credit rating class. Thus, in fact Carty and Fons [5] established, without stating it, that the appropriate model was not a simple Markov chain – in which case the duration of stay in each credit rating class would follow the exponential distribution (or the geometric distribution in the discrete case) – but rather was a semi-Markov process. There exists a generation of modeling dominated by duration analysis. The earliest in this literature is the work of Lane, Looney, and Wansley [28] on bank default prediction, using time-independent covariates. Lee and Urrutia [29] used a duration model based on a Weibull distribution of default time. Duration models on time-varying covariates include those of McDonald and Van de Gucht [33]. Related duration analysis by Shumway [40], Kavvathas [24], Chava and Jarrow [6] and Hillegeist, Keating, Cram, and Lundstedt [18] predict bankruptcy. Lately, inhomogeneous semi-Markov processes have been proposed for the migration process as more realistic models for the variability of the rating transition matrices by Vasileiou