

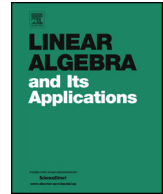


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A conjecture on the primitive degree of tensors [☆]



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ABSTRACT

In this paper, we prove the following: Let \mathbb{A} be a nonnegative primitive tensor with order m and dimension n . Then its primitive degree $\gamma(\mathbb{A}) \leq (n-1)^2 + 1$, and the upper bound is sharp. This confirms a conjecture of Shao [5].

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1. Introduction

In [1] and [2], Chang et al. investigated the properties of the spectra of nonnegative tensors. They defined the irreducibility of tensors, and the primitivity of nonnegative

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tensors, and extended many important properties of primitive matrices of primitive tensors. Recently, as an application of the general tensor product defined by Shao [5], Shao presented a simple characterization of the primitive tensors in terms of the zero pattern of the powers of \mathbb{A} . He also proposed the following conjecture on the primitive degree $\gamma(\mathbb{A})$.

Conjecture 1.1. *When m is fixed, then there exists some polynomial $f(n)$ on n such that $\gamma(\mathbb{A}) \leq f(n)$ for all nonnegative primitive tensors of order m and dimension n .*

In this paper, we confirm the conjecture by proving the following theorem.

Theorem 1.2. *Let \mathbb{A} be a nonnegative primitive tensor with order m and dimension n . Then its primitive degree $\gamma(\mathbb{A}) \leq (n-1)^2 + 1$, and the upper bound is sharp.*

2. Preliminaries

An order m and dimension n tensor $\mathbb{A} = (a_{i_1 i_2 \dots i_m})_{1 \leq i_j \leq n \ (j=1, \dots, m)}$ over the complex field \mathbb{C} is a multidimensional array with all entries $a_{i_1 i_2 \dots i_m} \in \mathbb{C}$ ($i_1, \dots, i_m \in [n] = \{1, \dots, n\}$). The majorization matrix $M(\mathbb{A})$ of the tensor \mathbb{A} is defined as $(M(\mathbb{A}))_{ij} = a_{ij \dots j}$ ($i, j \in [n]$) by Pearson [3].

Let \mathbb{A} (and \mathbb{B}) be an order $m \geq 2$ (and $k \geq 1$), dimension n tensor, respectively. Recently, Shao [5] defined a general product $\mathbb{A}\mathbb{B}$ to be the following tensor \mathbb{D} of order $(m-1)(k-1) + 1$ and dimension n :

$$d_{i\alpha_1 \dots \alpha_{m-1}} = \sum_{i_2, \dots, i_m=1}^n a_{ii_2 \dots i_m} b_{i_2 \alpha_1} \dots b_{i_m \alpha_{m-1}} \quad (i \in [n], \alpha_1, \dots, \alpha_{m-1} \in [n]^{k-1}).$$

The tensor product possesses a very useful property: the associative law [5, Theorem 1.1]. With the general product, Shao [5] proved some results on the primitivity and primitive degree of nonnegative tensors. The following result will be used in Definition 2.3.

Proposition 2.1. (See [5, Proposition 4.1].) *Let \mathbb{A} be an order m and dimension n nonnegative tensor. Then the following three conditions are equivalent:*

- (1). *For any $i, j \in [n]$, $a_{ij \dots j} > 0$ holds.*
- (2). *For any $j \in [n]$, $\mathbb{A}e_j > 0$ holds (where e_j is the j th column of the identity matrix I_n).*
- (3). *For any nonnegative nonzero vector $x \in \mathbb{R}^n$, $\mathbb{A}x > 0$ holds.*

Definition 2.2. (See [3, Definition 3.1].) A nonnegative tensor \mathbb{A} is called essentially positive, if it satisfies (3) of Proposition 2.1.

By Proposition 2.1, the following Definition 2.3 is equivalent to Definition 2.2.

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