

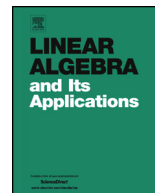


ELSEVIER

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa



Chain of evolution algebras of “chicken” population



U.A. Rozikov*, Sh.N. Murodov

Institute of Mathematics, 29, Do'rmon Yo'li str., 100125, Tashkent, Uzbekistan

ARTICLE INFO

Article history:

Received 30 September 2013

Accepted 3 March 2014

Available online 25 March 2014

Submitted by R. Brualdi

MSC:

17D92

17D99

60J27

Keywords:

Evolution algebra

Time

Chapman–Kolmogorov equation

Property transition

Idempotent

Nilpotent

ABSTRACT

Recently Ladra and Rozikov introduced a notion of evolution algebra of a “chicken” population (EACP). The algebra is given by a rectangular matrix of structural constants. In this paper we introduce a notion of chain of evolution algebras of a “chicken” population (CEACP). The sequence of matrices of the structural constants for this CEACP satisfies an analogue of Chapman–Kolmogorov equation (with a specific multiplication defined for rectangular matrices). We give several examples (time homogeneous, time non-homogeneous, periodic, etc.) of such chains. We construct some periodic 3-dimensional CEACP which contains a continuum set of non-isomorphic EACP and show that the corresponding discrete time CEACP is dense in the set. Moreover we study time depending dynamics of 2 and 3 dimensional CEACP to be isomorphic to a fixed algebra.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

The action of genes is manifested statistically in sufficiently large communities of matching individuals (belonging to the same species). These communities are called populations.

* Corresponding author. Tel.: +998 712627544; fax: +998 712627357.

E-mail addresses: rozikovu@yandex.ru (U.A. Rozikov), sherruriy_1086@mail.ru (Sh.N. Murodov).

The evolution (or dynamics) of a population comprises a determined change of state in the next generations as a result of reproduction and selection. This evolution of a population can be studied by a dynamical system (iterations) of a quadratic stochastic operator (see review paper [6]).

Etherington introduced the formal language of abstract algebra to the study of genetics in a series of seminal papers [2–4]. In recent years many authors have tried to investigate the difficult problem of classification of these algebras.

Recently in the book of J.P. Tian [13] a new type of evolution algebra was introduced. This algebra also describes some evolution laws of genetics. The study of evolution algebras constitutes a new subject both in algebra and the theory of dynamical systems. In the book [13], the foundation of evolution algebra theory and applications in non-Mendelian genetics and Markov chains are developed.

In [1] a notion of chain of evolution algebras is introduced. The sequence of matrices of the structural constants for this chain of evolution algebras satisfies an analogue of Chapman–Kolmogorov equation. The basic properties of such chains are studied.

In the book [11] the evolution algebras associated with a free population are studied. But there are few results devoted to evolution algebras corresponding to bisexual populations.

Recently in [9] the authors considered a bisexual population and defined an evolution algebra using inheritance coefficients of the population. This algebra is a natural generalization of the algebra of free population. Moreover in [10] an evolution algebra of a “chicken” population (EACP) is considered. This algebra corresponds to a bisexual population with a set of females partitioned into finitely many different types and the males having only one type. The basic properties of the EACP are studied.

In this paper we introduce a notion of a chain of evolution algebras of “chicken” population (CEACP) and study its dynamical properties.

The paper is organized as follows. In Section 2 we give main definitions related to CEACP. In Section 3 we give several examples (time homogeneous, time non-homogeneous, periodic, etc.) of such chains. For a periodic chain of evolution algebras we construct a continuum set of non-isomorphic evolution algebras and show that the corresponding discrete time chain of evolution algebras is dense in the set. In Section 4 we study the time depending dynamics of 2 and 3 dimensional CEACP to be isomorphic to a fixed algebra.

2. Definition of CEACP

Following [10] we consider a set $\{h_i, i = 1, \dots, n\}$ (the set of “hen”s) and r (a “rooster”).

Definition 2.1. (See [10].) Let (\mathcal{E}, \cdot) be an algebra over a field K of characteristic $\neq 2$. If it admits a basis $\{h_1, \dots, h_n, r\}$, such that

Download English Version:

<https://daneshyari.com/en/article/4599600>

Download Persian Version:

<https://daneshyari.com/article/4599600>

[Daneshyari.com](https://daneshyari.com)