

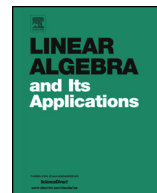


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The inertia of weighted unicyclic graphs<sup>☆</sup>Guihai Yu<sup>a,b</sup>, Xiao-Dong Zhang<sup>c,\*</sup>, Lihua Feng<sup>d</sup><sup>a</sup> School of Mathematics, Shandong Institute of Business and Technology, Yantai, Shandong, 264005, China<sup>b</sup> Center for Combinatorics, Nankai University, Tianjin, 300071, China<sup>c</sup> Department of Mathematics and MOE-LSC, Shanghai Jiao Tong University, Shanghai, 200240, China<sup>d</sup> Department of Mathematics, Central South University, Railway Campus, Changsha, Hunan, 410075, China

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## ABSTRACT

Let  $G_w$  be a weighted graph. The *inertia* of  $G_w$  is the triple  $In(G_w) = (i_+(G_w), i_-(G_w), i_0(G_w))$ , where  $i_+(G_w)$ ,  $i_-(G_w)$ ,  $i_0(G_w)$  are the numbers of the positive, negative and zero eigenvalues of the adjacency matrix  $A(G_w)$  of  $G_w$  including their multiplicities, respectively.  $i_+(G_w)$ ,  $i_-(G_w)$  are called the *positive*, *negative indices of inertia* of  $G_w$ , respectively. In this paper we present a lower bound for the positive, negative indices of weighted unicyclic graphs of order  $n$  with fixed girth and characterize all weighted unicyclic graphs attaining this lower bound. Moreover, we characterize the weighted unicyclic graphs of order  $n$  with two positive, two negative and at least  $n - 6$  zero eigenvalues, respectively.

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\* Corresponding author.

E-mail addresses: [yuguihai@126.com](mailto:yuguihai@126.com) (G. Yu), [xiaodong@sjtu.edu.cn](mailto:xiaodong@sjtu.edu.cn) (X.-D. Zhang), [fenglh@163.com](mailto:fenglh@163.com) (L. Feng).

## 1. Introduction

Let  $G$  be a simple graph of order  $n$  with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$  and edge set  $E(G)$ . The *adjacency matrix*  $A(G) = (a_{ij})$  of graph  $G$  of order  $n$  is a symmetric  $(0, 1)$ -matrix such that  $a_{ij} = 1$  if  $v_i$  is adjacent to  $v_j$  and 0 otherwise. A weighted graph  $G_w$  is a pair  $(G, w)$  where  $G$  is a simple graph with edge set  $E(G)$ , called the *underlying graph* of  $G_w$ , and  $w$  is a weight function from  $E(G)$  to the set of nonzero real numbers. The *adjacency matrix* of  $G_w$  on  $n$  vertices is defined as the matrix  $A(G_w) = (a_{ij})$  such that  $a_{ij} = w(v_i v_j)$  if  $v_i$  is adjacent to  $v_j$  and 0 otherwise. The characteristic polynomial of  $G_w$  is the characteristic polynomial of  $A(G_w)$ , denoted by

$$P_{G_w}(\lambda) = \det(\lambda I - A(G_w)) = \lambda^n + a_1^* \lambda^{n-1} + \dots + a_n^*.$$

The *inertia* of  $G_w$  is defined to be the triple  $In(G_w) = (i_+(G_w), i_-(G_w), i_0(G_w))$ , where  $i_+(G_w)$ ,  $i_-(G_w)$ ,  $i_0(G_w)$  are the numbers of the positive, negative and zero eigenvalues of  $A(G_w)$  including multiplicities, respectively.  $i_+(G_w)$  and  $i_-(G_w)$  are called the *positive, negative indices of inertia* (abbreviated *positive, negative indices*) of  $G_w$ , respectively. The number  $i_0(G_w)$  is called the *nullity* of  $G_w$ . The rank of an  $n$ -vertex graph  $G_w$ , denoted by  $r(G_w)$ , is defined as the rank of  $A(G_w)$ . Obviously,  $r(G_w) = i_+(G_w) + i_-(G_w) = n - i_0(G_w)$ .

A graph  $G_w$  is called *acyclic* (resp. *unicyclic*, *bipartite*) if its underlying graph  $G$  is *acyclic* (resp. *unicyclic*, *bipartite*). An *induced subgraph* of  $G_w$  is an induced subgraph of  $G$  with the same weights. For a subgraph  $H_w$  of  $G_w$ , let  $G_w - H_w$  be the subgraph obtained from  $G_w$  by deleting all vertices of  $H_w$  and all incident edges. For  $V' \subseteq V(G_w)$ ,  $G_w - V'$  is the subgraph obtained from  $G_w$  by deleting all vertices in  $V'$  and all their incident edges. A vertex of a graph  $G_w$  is called *pendant* if it has degree one, and is called *quasi-pendant* if it is adjacent to a pendant vertex. For a weighted graph  $G_w$  on at least two vertices, a vertex  $v \in V(G_w)$  is called *unsaturated* in  $G_w$  if there exists a maximum matching  $M$  of  $G$  in which no edge is incident with  $v$ ; otherwise,  $v$  is called *saturated* in  $G_w$ .

A simple graph may be regarded as a weighted graph in which the weight of each edge is  $+1$ . A signed graph may be regarded as a weighted graph in which the weight of each edge is  $+1$  or  $-1$ . Moreover, the sign of a signed cycle, denoted by  $sgn(C)$ , is defined as the sign of the product of all edge weights  $+1$  or  $-1$  on  $C$ . The signed cycle  $C$  is said to be *positive* (or *negative*) if  $sgn(C) = +$  (or  $sgn(C) = -$ ). A signed graph is said to be *balanced* if all its cycles are positive, otherwise it is called *unbalanced*.

The study of eigenvalues of a weighted graph has attracted much attention. Several results about the (Laplacian) spectral radius of weighted graphs were derived in [1,10,9,24,25]. The inertia of unweighted graphs has attracted some attention. Gregory et al. [17] studied the subadditivity of the positive, negative indices of inertia and developed certain properties of Hermitian rank which were used to characterize the biclique decomposition number. Gregory et al. [16] investigated the inertia of a partial join of two graphs and

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