

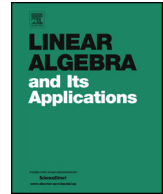


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# Linear Algebra and its Applications

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## The minimum rank of a sign pattern matrix with a 1-separation



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### ABSTRACT

A sign pattern matrix is a matrix whose entries are from the set  $\{+, -, 0\}$ . If  $A$  is an  $m \times n$  sign pattern matrix, the qualitative class of  $A$ , denoted  $Q(A)$ , is the set of all real  $m \times n$  matrices  $B = [b_{i,j}]$  with  $b_{i,j}$  positive (respectively, negative, zero) if  $a_{i,j}$  is  $+$  (respectively,  $-$ ,  $0$ ). The minimum rank of a sign pattern matrix  $A$ , denoted  $\text{mr}(A)$ , is the minimum of the ranks of the real matrices in  $Q(A)$ . Determination of the minimum rank of a sign pattern matrix is a longstanding open problem.

For the case that the sign pattern matrix has a 1-separation, we present a formula to compute the minimum rank of a sign pattern matrix using the minimum ranks of certain generalized sign pattern matrices associated with the 1-separation.

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## 0. Introduction

A *sign pattern matrix* (or *sign pattern*) is a matrix whose entries are from the set  $\{+, -, 0\}$ . If  $B = [b_{i,j}]$  is a real matrix, then  $\text{sgn}(B)$  is the sign pattern matrix  $A = [a_{i,j}]$  with  $a_{i,j} = +$  (respectively,  $-$ ,  $0$ ) if  $b_{i,j}$  is positive (respectively, negative, zero). If  $A$  is a sign pattern matrix, the *sign pattern class* of  $A$ , denoted  $Q(A)$ , is the set of all real matrices  $B = [b_{i,j}]$  with  $\text{sgn}(B) = A$ . The *minimum rank* of a sign pattern matrix  $A$ , denoted  $\text{mr}(A)$ , is the minimum of the ranks of matrices in  $Q(A)$ ; see [5]. Recently, Li et al. [7] obtained a characterization of sign pattern matrices  $A$  with  $\text{mr}(A) \leq 2$ . In this paper, we present a formula to compute the minimum rank of a sign pattern matrix with a 1-separation using the minimum ranks of certain generalized sign pattern matrices associated with the 1-separation.

The notion of sign pattern matrix can be extended to generalized sign pattern matrices by allowing certain entries to be  $\#$ ; see [5]. For a generalized sign pattern matrix  $A$ , the generalized sign pattern class of  $A$ , denoted  $Q(A)$ , is defined by allowing entries of a matrix  $B = [b_{i,j}] \in Q(A)$  to be any real number if the corresponding entries of  $A$  are  $\#$ . The minimum rank  $\text{mr}(A)$  of a generalized sign pattern matrix  $A$  is defined in the same way as for a sign pattern matrix:  $\text{mr}(A)$  is the minimum of the ranks of matrices in  $Q(A)$ . If  $A = [a_{i,j}]$  and  $C = [c_{i,j}]$  are generalized sign pattern matrices of the same size, we write  $A \leq C$  if for each entry of  $A$ ,  $a_{i,j} = c_{i,j}$  or  $c_{i,j} = \#$ . It is clear that if  $A \leq C$ , then  $Q(A) \subseteq Q(C)$ . For a generalized sign pattern  $C$ , let  $\mathcal{C}$  be the set of all sign pattern matrices  $A$  such that  $A \leq C$ . Then, clearly,  $Q(C) = \bigcup_{A \in \mathcal{C}} Q(A)$ . Hence the minimum rank of a generalized sign pattern matrix  $C$  equals  $\min_{A \in \mathcal{C}} \text{mr}(A)$ .

We define subtraction of two elements from  $\{+, -, 0\}$  as follows:

1.  $(+) - (0) = +$ ,  $(0) - (-) = +$ ,  $(+) - (-) = +$ ,
2.  $(-) - (+) = -$ ,  $(0) - (+) = -$ ,  $(-) - (0) = -$ ,
3.  $(0) - (0) = 0$ ,
4.  $(+) - (+) = \#$ ,  $(-) - (-) = \#$ .

The idea behind the definition of, for example,  $(-) - (+) = -$  is that subtracting a positive number from a negative number gives a negative number.

Let

$$M = \begin{bmatrix} A_{1,1} & A_{1,2} & 0 \\ A_{2,1} & a_{2,2} & A_{2,3} \\ 0 & A_{3,2} & A_{3,3} \end{bmatrix}$$

be a sign pattern matrix, where  $A_{1,2}$  and  $A_{3,2}$  have only one column, and  $A_{2,1}$  and  $A_{2,3}$  have only one row. We also say that the sign pattern matrix  $M$  has a 1-separation. For  $p \in \{+, -, 0\}$ , let

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