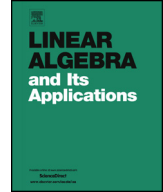




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Complete multipartite graphs are determined by their distance spectra [☆]



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ABSTRACT

In this paper, we prove that the complete multipartite graphs are determined by their distance spectra, which confirms the conjecture proposed by Lin, Hong, Wang and Shu (2013) [7], although it is well known that the complete multipartite graphs cannot be determined by their adjacency spectra.

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1. Introduction

In this paper, we only consider simple and undirected connected graphs. Let $G = (V(G), E(G))$ be a graph of order n with vertex set $V(G)$ and edge set $E(G)$. Let $d_G(v)$ and $N_G(v)$ denote the degree and neighbors of a vertex v , respectively. $D(G) = (a_{uv})_{n \times n}$

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denotes the *distance matrix* of G with $a_{uv} = d_G(u, v)$, where $d_G(u, v)$ is the distance between vertices u and v . The largest eigenvalue of $D(G)$, denoted by $\lambda(G)$, is called the *distance spectral radius* of G . The research for distance matrix can be dated back to the paper [6,5], which presents an interesting result that the determinant of the distance matrix of trees with order n is always $(-1)^{n-1}(n-1)2^{n-2}$, independent of the structure of the tree. Recently, the distance matrix of a graph has received increasing attention. For example, Liu [8] characterized the graphs with minimal spectral radius of the distance matrix in three classes of simple connected graphs with fixed vertex connectivity, matching number and chromatic number, respectively. Zhang [10] determined the unique graph with minimum distance spectral radius among all connected graphs with a given diameter. Bose, Nath and Paul [1] characterized the graph with minimal distance spectral radius among all graphs with the fixed number of pendent vertices.

Denote by $Sp(D(G))$ the set of all eigenvalues of $D(G)$ including the multiplicity. Two graphs G and G' are called *D-cospectral* if $Sp(D(G)) = Sp(D(G'))$. A graph G is determined by its D-spectra if $Sp(D(G)) = Sp(D(G'))$ implies $G \cong G'$. There are three excellent surveys [2–4] on that which graphs can be determined by their spectra. Lin et al. [7] characterized all the connected graphs with smallest eigenvalue being -2 and proposed the following conjecture:

Conjecture 1.1. (See [7].) *Let $G = K_{n_1, n_2, \dots, n_k}$ be a complete k -partite graph. Then G is determined by its D-spectrum.*

Moreover, they proved that the conjecture is true for $k = 2$. On the other hand, it is well known that complete multipartite graphs cannot be determined by their spectra. For example, $K_{1,4}$ is not determined by its adjacency spectra, since $K_{1,4}$ and the union of cycle of order 4 and an isolated vertex have the same spectrum but are not isomorphic. In this paper, we will give a positive answer to Conjecture 1.1.

2. Main results

Before presenting the proof of the conjecture, we need the following theorems and lemmas.

Theorem 2.1. (See [7].) *Let $D(G)$ be the distance matrix of a connected graph G . Then the smallest eigenvalue of $D(G)$ is equal to -2 with multiplicity $n - k$ if and only if G is a complete k -partite graph for $2 \leq k \leq n - 1$.*

Theorem 2.2. (See [7].) *Let $G = K_{n_1, \dots, n_k}$ be a complete k -partite graph. Then the characteristic polynomial of $D(G)$ is*

$$P_D(\lambda) = (\lambda + 2)^{n-k} \left[\prod_{i=1}^k (\lambda - n_i + 2) - \sum_{i=1}^k n_i \prod_{j=1, j \neq i}^k (\lambda - n_j + 2) \right].$$

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