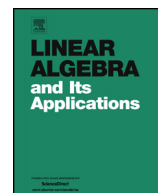




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Counterexamples to an edge spread question for zero forcing number



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ABSTRACT

This note gives counterexamples to an edge spread problem on the zero forcing number.

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1. Introduction

For a simple graph G on n vertices, the *minimum rank problem* is to determine the *minimum rank* $\text{mr}(G)$ of G , which is the smallest possible rank among all $n \times n$ symmetric

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matrices whose off-diagonal ij -entry is nonzero whenever ij is an edge in G and zero otherwise. Equivalently, we may consider the *maximal nullity* $M(G) := n - \text{mr}(G)$. In 2008, a graph parameter called the *zero forcing number* was introduced by the AIM Minimum Rank-Special Graphs Work Group [1] to serve as an upper bound for the maximal nullity.

Let G be a graph with each vertex colored black or white. A *zero forcing process* on G is defined by the *color-change rule*: a black vertex x can force a white vertex y to black at the next step when y is the only white neighbor of x . A *zero forcing set* of G is a subset $B \subseteq V(G)$ which as the initial set of black vertices can force all vertices in $V(G)$ to black. The *zero forcing number* $Z(G)$ is the minimum size of a zero forcing set of G . It was proved in [1] that

$$M(G) \leq Z(G) \quad \text{for any graph } G.$$

To describe a zero forcing process, we write $x \rightarrow y$ to denote that at some stage a black vertex x forces its only white neighbor y to black. The *chronological list* of a zero forcing process is a record $(x_i \rightarrow y_i; 1 \leq i \leq h)$ in order. Thus when we mention a zero forcing process, we mean the corresponding zero forcing set together with its chronological list.

A *maximal chain* is a maximal sequence of vertices of the form

$$x_1 \rightarrow x_2 \rightarrow \cdots \rightarrow x_k.$$

It is worth noting that every maximal chain is an induced path in G , by the color-change rule. Also, the set of all maximal chains of a zero forcing process is a collection of induced paths that cover all vertices of the graph.

The *edge spread* $z_e(G)$ of a graph G on an edge e is

$$z_e(G) = Z(G) - Z(G - e).$$

Edholm et al. [2] proved that if $z_e(G) = -1$, then the edge e is an edge in some maximal chain of every optimal zero forcing process. Here an optimal zero forcing process means one whose corresponding zero forcing set is of size $Z(G)$. In the same paper, the authors asked in Question 2.22 if the converse of the previous statement is also true. The purpose of this note is to give a negative answer to the question by constructing infinitely many counterexamples.

2. The robot graphs

The counterexamples provided in this note are the *robot graphs* $R_n(h_1, h_2, \dots, h_{2n})$ to be constructed as follows. Choose integers $n \geq 2$, $h_0 = 2$ and $h_s \geq 3$ for $1 \leq s \leq 2n$. Let P_s be the path with the h_s vertices labeled $p_{s,1}, p_{s,2}, \dots, p_{s,h_s}$ for $0 \leq s \leq 2n$; and let C_{2n+1} be the cycle with the $2n+1$ vertices labeled $v_0, v_1, v_2, \dots, v_{2n}$. Define H_p to be the graph obtained from $C_{2n+1}, P_0, P_1, \dots, P_{2n}$ by identifying p_{s,h_s} with v_s for $0 \leq s \leq 2n$,

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