Linear Algebra and its Applications 446 (2014) 388–420

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Miniversal deformations of matrices under *congruence and reducing transformations

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ARTICLE INFO

Article history:
Received 26 September 2013
Accepted 13 January 2014
Available online 30 January 2014
Submitted by F. Dopico

MSC:
15A21
15A63
47A07

Keywords:
Sesquilinear forms
*Congruence transformations
Miniversal deformations

ABSTRACT

Arnold (1971) [1] constructed a miniversal deformation of a square complex matrix under similarity; that is, a simple normal form to which not only a given square matrix A but all matrices B close to it can be reduced by similarity transformations that smoothly depend on the entries of B. We give miniversal deformations of matrices of sesquilinear forms; that is, of square complex matrices under *congruence, and construct an analytic reducing transformation to a miniversal deformation. Analogous results for matrices under congruence were obtained by Dmytryshyn, Futorny, and Sergeichuk (2012) [11].

1. Introduction

The reduction of a matrix to its Jordan form is an unstable operation: both the Jordan form and the reduction transformation depend discontinuously on the entries of

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0024-3795/$ – see front matter
http://dx.doi.org/10.1016/j.laa.2014.01.016

the original matrix. Therefore, if the entries of a matrix are known only approximately, then it is unwise to reduce it to Jordan form.

Furthermore, when investigating a family of matrices close to a given matrix, then although each individual matrix can be reduced to a Jordan form, it is unwise to do so since in such an operation the smoothness relative to the entries is lost.

For these reasons, Arnold [1, Theorem 4.4] (see also [2,3]) constructed a *miniversal deformation* of each Jordan matrix J; that is, a simple normal form to which all matrices A close to J can be reduced by similarity transformations that smoothly depend on the entries of A.

Miniversal deformations were also constructed

- for real matrices with respect to similarity by Galin [16] (see also [2,3]) and simplified by Garcia-Planas and Sergeichuk [19];
- for complex matrix pencils by Edelman, Elmroth, and Kågström [14]; for complex and real matrix pencils and contragredient matrix pencils (i.e., pairs of linear maps $U \rightleftarrows V$) by Garcia-Planas, Klimenko, and Sergeichuk [19,26];
- for matrices up to congruence by the authors [11]; for pairs of skew-symmetric or symmetric matrices up to congruence by Dmytryshyn [9,10];
- for matrices up to unitary similarity by Benedetti and Cragnolini [5]; for matrices of selfadjoint operators on a complex or real vector space with scalar product given by a skew-symmetric, or symmetric, or Hermitian nonsingular form in [8,17,33,34]. Deformations of Hermitian matrices were studied by von Neumann and Wigner [43].

An informal introduction to perturbations of matrices determined up to similarity, congruence, or *congruence is given by Klimenko and Sergeichuk [27].

All matrices that we consider are complex matrices.

The main results of this paper are the following:

- we construct a miniversal deformation of a square complex matrix A with respect to **congruence transformations* S^*AS (S is nonsingular); i.e., we give a simple normal form B_{mindef} to which all matrices B close to A can be reduced by a *congruence transformation that is an analytic function of the entries of both B and its complex conjugate $\bar{B}$;
- we construct this analytic *congruence transformation.

Applications of Arnold's miniversal deformations of matrices under similarity are based, in particular, on the fact that a matrix and its Arnold normal form have the same eigenvalues [28–30]. In a similar way, possible applications of the normal form B_{mindef} can be based on the fact that B and B_{mindef} have the same invariants with respect to *congruence, while B_{mindef} has a very simple structure. A preliminary version of this article was used in [15] for constructing the Hasse diagram of the closure ordering on the set of *congruence classes of 2×2 matrices.